Volume XVI. *March, 1903.* *Number 3.*

THE

PHYSICAL REVIEW.

THE ULTRA-VIOLET REFLECTING POWER OF SELENIUM, CYANINE AND GLASS.

By P. G. Nutting.

WITHIN the past two years the reflecting power of substances for the shorter wave-length radiation has been attacked by several observers. Glatzel[1] investigated the reflecting power of silver, iron, nickel, copper and some alloys. His method was to reflect two beams of light, one from a known standard mirror and one from the mirror being investigated, on the two halves of a double slit and then vary the width of one half the slit until the light from the two halves produced the same photographic impression. His standard mirror was of Brandes-Schuneman alloy and its reflecting power, exterpolated from observed values in the visible spectrum was considered constant in the ultra-violet. The author[2] observed the reflecting power of a number of metals and organic liquids by finding the time of exposure necessary to give the spectrum reflected from these substances the same photographic intensity as that from quartz, the reflecting power of quartz being calculated from its known refractive indices. All of the above-mentioned work was however of but little quantitative value and it seemed desirable to repeat it by some accurate photometric method.

[1] Physik. Zeit., 2, 173, 1900.
[2] Physical Review, 13, 193, 1901

THE PHOTOGRAPHIC POLARIZATION PHOTOMETER.

Three improvements in the accuracy of photometric methods have been made in recent years. Wild[1] polarized the two beams whose intensities were to be made equal in directions normal to each other and then brought them through a quartz wedge polariscope to overlapping. In this way the two photometric fields are divided into narrow bands lying alternately and the eye is able to detect much smaller differences than in the case of ordinary photometers. Simon[2] allowed the two beams whose intensities were to be compared to fall on the two halves of a double slit and drew a photographic plate slowly past the slit which was gradually varied in width. The two bands thus produced on the plate would show at some point equal intensity and from the relative widths of the two halves of the slit at this point, the relative intensities of the two beams of light could be calculated. Thus by the application of photography, a means of interpolation is obtained which is impossible in direct eye observations. In the present investigation photography was applied to a polarization method somewhat similar to that of Wild.

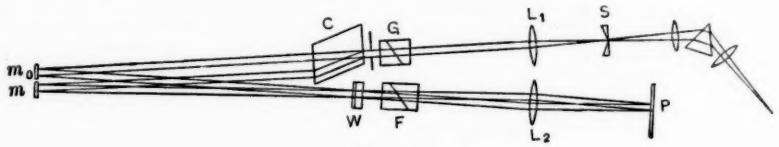

Fig. 1.

A lens L_1 gives an image of the slit S at the point W, monochromatic light from a quartz spectrometer being used. The large calcite crystal C divides the beam of light into two, polarized in and normal to the (horizontal) plane of the apparatus. These two rays are reflected on the two mirrors m_0 and m at nearly normal incidence. These two mirrors are slightly inclined to each other so that the two rays cross at the center of the wedge pair W and continuing divergent, pass through the nicol F and are brought together a second time by the lens L_2 upon the photographic plate P. On the plate then are two superposed images of the slit, each

[1] Pogg. Ann., 118, 193, 1863.
[2] Wied. Ann., 59, 91, 1896.

crossed by interference bands produced by the wedge pair W. A perforated screen just in front of the crystal C makes sure that the two beams do not overlap on the mirrors. The distance SL_1 is made small and L_1W great so that the light between L_1 and W is a bundle of very nearly parallel rays. The whole distance L_1mW is about 200 cm. The large crystal C is about 60 mm. thick, so that for yellow light the separation of the two rays is about 7 mm. The angle of incidence on the mirrors is less than three degrees, so that the rotation produced by even metallic mirrors is quite negligible. The two mirrors are inclined to each other at an angle of about thirty minutes. The slit opening used is about 0.2 by 1.0 millimeter and the beams of light at the mirrors only about 3 by 10 millimeters so that very small mirrors may be used. These mirrors must, however, have good optical surfaces, else the images at W and P will not be uniformly bright. The phase wedges W were specially cut by Messrs. Steeg and Reuter with the wedge edges making angles of 45° with the optical axis of the quartz. Such wedges may be placed with either side or end parallel to the plane of polarization of the light used and still give bands parallel to the wedge edge. The wedges were put together with glycerine instead of Canada balsam. The mirrors were mounted like Fresnel mirrors on a specially designed brass table. This table was so constructed that the two mirrors could be easily interchanged by inverting the whole support without disturbing the adjustments.

Now each of the two rays gives at P an image filled with horizontal light and dark bands, but since their planes of polarization differ in azimuth by 90° as the rays pass into the wedge pair, the light bands in the image given by one ray exactly overlap the dark bands in the image from the other ray. When the two rays are of equal intensity, the bands disappear entirely from the resultant image. Their relative intensity is varied by rotating a Glan nicol G in the path of the ray before it enters the large crystal. If now the reflecting power of one of the mirrors is known, the reflecting power of the other may be calculated from the rotation of the nicol G from the zero position. The mirror of known reflecting power was of quartz cut with parallel faces parallel to the optic axis so that its reflecting power could be calculated from its refractive indices by

means of the formulas of Franz Neumann. The refractive indices have been very accurately determined by Sarasin, Simon and Trommsdorff. In practice an exposure is made with rotatable nicol nearly in the position for which the bands disappear, then the plate displaced a few millimeters, the nicol turned, say one degree, a second exposure made and the process repeated until perhaps six exposures have been made side by side on the same plate. Among these six will be one perhaps which shows no trace of bands, or an intermediate position may be estimated for which none would appear. The ordinary working sensitiveness was usually about one part in a thousand for the difference in the relative intensity of the two rays from unity. If all parts of the instrument were specially constructed for this particular use, this form of photometer ought easily to give results to a hundredth of one per cent. so that it would be by far the most accurate known.

From various causes, chiefly reflection on the various crystal surfaces, the two rays suffer other unequal losses than that at the mirrors. Let K be the ratio of the connection factors for the two rays. Then we have for calculating the unknown reflecting power

$$R = R_0 K \tan^2 \theta$$

where θ is the angle through which the balancing nicol is turned from its zero position. Exchange the two mirrors and take a new reading and we have

$$R_0 = RK \tan^2 \theta'$$

and by eliminating the unknown correction factor R,

$$R = R_0 \tan \theta \cot \theta'$$

This method of exchanging the mirrors to eliminate experimentally all corrections was used throughout all the work.

As a source of radiation, the Nernst lamp was used in the visible and sparks of aluminum, zinc and cadmium in the ultra-violet. The work was usually carried out as far as the aluminum line at 236 $\mu\mu$. Between 230 and 200 $\mu\mu$, calcite is strongly dichroic. For the ordinary ray the absorption is practically complete beyond 220 $\mu\mu$, for the extraordinary ray beyond 195 $\mu\mu$, but the absorption begins

very gradually, not abruptly as in the case of glass. The time of exposure necessary varied from ten seconds to ten minutes according to the intensity of the spectrum line used and the reflecting power of the mirror being investigated.

Measurements.

The first determinations of reflecting power were made on a steel mirror prepared by Zeiss. The results are shown in the table of data and the accompanying curve. There is a decided shoulder in the curve between 309 and 334 $\mu\mu$, otherwise the reflecting power falls off quite regularly from 56 per cent. in the yellow to about 33 per cent. at 257 $\mu\mu$. Hagen and Rubens have determined the reflecting power of steel and a number of other metals in the ultra-violet and infra-red[1] as well as in the visible spectrum,[2] and their work in the ultra-violet appearing about the time the above determinations on steel were completed, the author turned his attention to those non-metals which show metallic absorption at certain wave-lengths.

Reflecting Power of Steel.

Wave-length,	589	534	470	410	361	334	309	274	257
Ref. power,	55.9	55.4	54.0	52.0	48.8	43.7	43.0	35.3	33.3

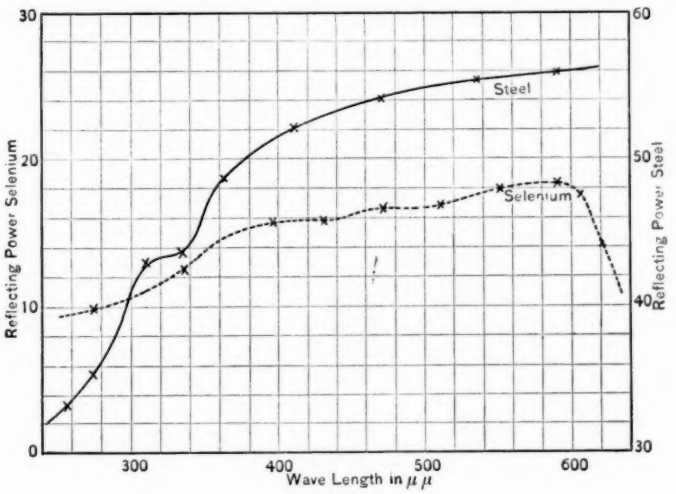

Fig. 2.

[1] Drude's Ann., 8, 1–21, 1902.

[2] Zeit. f. Instrumentenkunde, 19, 305, 1899.

Selenium.—In a great many of its physical properties, selenium occupies a position midway between the metals and the non-metals. In its optical properties it is particularly interesting. Its strong and slowly varying absorbing power in the visible and ultra-violet, as well as its large reflecting power, would lead to its classification with the metals, while its transparency over so wide a region in the infra-red is just as strongly characteristic of the non-metals. The reflective and absorptive indices have been determined in the visible spectrum by Quincke [1] and in the visible and ultra-violet by Cornu [2] and by Wood.[3] Excellent mirrors of selenium were obtained by fusing it on ground glass, covering with plate glass and, when cold, removing this plate. The observed reflecting power increases abruptly from a low value in the red to a maximum in the yellow, falls off slightly toward the violet and then more rapidly in the ultra-violet. These results are not concordant with those of Wood. If we calculate the reflecting power from his refractive and absorptive indices by means of Drude's reflection formula

$$R = \frac{n^2(1 + \varkappa^2) + 1 - 2n}{n^2(1 + \varkappa^2) + 1 + 2n},$$

we obtain a curve rising rapidly from the yellow to the violet and then rising more slowly in the ultra-violet. Cornu observed at but three wave-lengths: 721, 439 and 280 $\mu\mu$; but the reflecting powers calculated from his values of the principal incidence and ellipticity agree roughly with those observed.

Reflecting Power of Selenium.

Wave-length,	620	605	589	550	510	470	430	395	334	274
Ref. power,	14.34	17.63	18.39	18.01	16.85	16.67	15.83	15.14	12.65	9.98

Only the last figures of the reflecting powers quoted are considered uncertain as regards error in measurement. This is the most accurate work done with the spectrometer and shows what work it will do when carefully adjusted, working with substances of sufficient reflecting power.

[1] Wied. Ann., Jubelband, p. 336, 1874.
[2] Compt. Rend., 108, 917 and 1211, 1889.
[3] Phil. Mag. (6), 3, 607, June, 1902.

Cyanine. — With its strong, narrow absorption band in the yellow, cyanine is an ideal substance for studying the anomalous dispersion of solids. Its refracting and absorbing power have been investigated by Pflüger[1] and its refracting power by Wood and Magnusson.[2] The cyanine used was the di-amyl iodide, $C_{29}H_{35}N_2I$, furnished by Kahlbaum, easily soluble in alcohol, but only very slightly soluble in water. Excellent mirrors are easily obtained by casting, great care being exercised in guarding against burning and keeping free from impurities. Fused cyanine appears to be an excellent solvent for grease and many other organic substances and requires the greatest care in handling if it is to be kept pure. The reflecting power rises in the red to a maximum in the yellow, falls to 1.26 per cent. in the green and then gradually increases to a nearly constant value of 6.8 per cent. in the ultra-violet. The reflecting power curve calculated from Wood and Magnusson's refractive indices and Pflüger's absorptive indices by Drude's formula agrees closely in form with the observed. Using the observed reflecting powers and the refractive indices determined by Wood and Magnusson, the absorptive indices were calculated and are given in the table.

Optical Constants of Cyanine.

Wave-length.	Reflecting Power.		n (W. & M.)	κ (calc.)
	Fresh.	Exposed.		
630	13.46		2.325	
610	14.22		2.120	0.192
589	13.31	6.25	1.660	0.555
560	10.50		1.250	0.954
540	7.81		1.125	0.538
520	2.21		1.130	0.257
500	1.26	6.14	1.255	0.070
475	2.04		1.400	0.00
450	3.42	6.57	1.490	
425		7.48	1.538	
395	5.68	7.8	1.578	
358	6.60			
334	6.94	6.9		
309	6.77			
274	6.77	6.5		

[1] Wied. Ann., 65, 173, 1898.
[2] Phil. Mag. (6), 1, 36, 1901

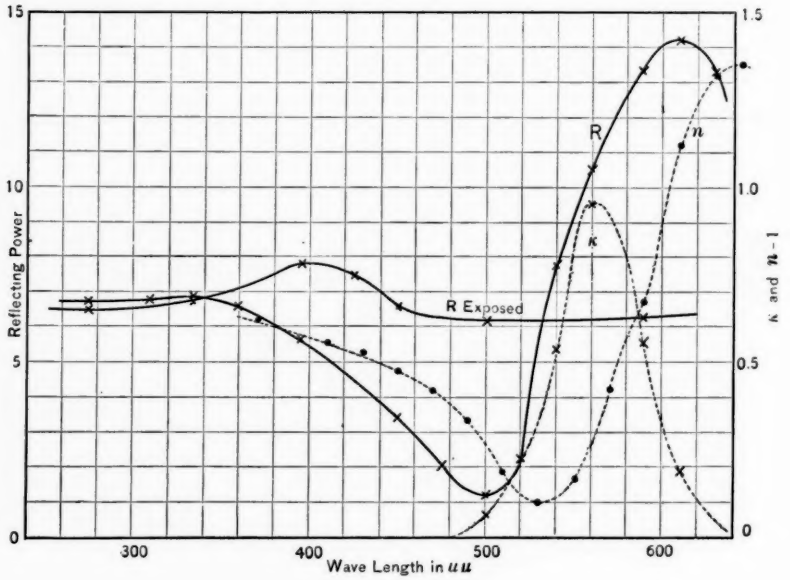

Fig. 3.

Exposure to light produces great changes in the reflecting power and other optical constants of cyanine in the visible spectrum. Freshly fused cyanine is of a deep brassy yellow color, but exposure for a few minutes to direct sunlight or for an hour to the dull light of a cloudy day turns it to a decided plum color. Exposed ten times as long in the same manner, it turns a steely blue-black and has the appearance of selenium. The change is not due to the heating effect of the absorbed radiation, for a piece of fresh cyanine kept warm in the dark shows no trace of change. On the other hand, a thin wash of cyanine on ground glass, exposed to light until well colored, recovers its brassy yellow color on refusion. An attempt to photograph the spectrum of a Nernst lamp on a cyanine mirror (exposure twelve hours), showed the effect to be strong in the yellow and but slight in the adjacent red and green. No trace of change could be detected in the remainder of the visible or ultra-violet spectra. The reflecting power curve for exposed cyanine shows a remarkable change to have taken place. Instead of a reflecting power of 14 per cent. in the yellow and 1.26 per cent. in the green, we have a constant reflecting power of 6.5 per cent.

through both regions and this is almost exactly the same as the constant reflecting power in the ultra-violet. The strong, characteristic anomalous dispersion of the cyanine has entirely disappeared. If we regard the strong absorption for yellow light as due to resonance, as is indicated by the anomalous dispersion, the effect of the light is to fatigue or destroy this resonance, warming restores it. The effect of this loss of resonating power on the absorptive and refractive indices is enormous. For example at 500 $\mu\mu$, where the absorption is so slight we may calculate the refractive index directly from the reflecting power, exposure to light increases the refractive index from 1.25 to 1.66. Evidently no reliable work on the dispersion of cyanine can be carried on where the prism or film is exposed to daylight. Again, in the yellow at about 580 $\mu\mu$ the refractive index of the exposed is the same as that of fresh cyanine. Here then the absorptive index has dropped from 0.70 to less than 0.7. This change in the absorption is easily seen by viewing a sodium flame though a piece of ground glass washed with fused cyanine. Through the fresh cyanine the flame is invisible but it is plainly seen through the cyanine exposed to light.

Glass. — The absorption of ordinary crown and flint glass is very strong in the ultra-violet and begins very abruptly, so that we might expect traces of anomalous dispersion in this region. To test whether the reflecting power showed any abrupt increase and to measure the absorptive index in this region, samples of standard Jena glass were kindly furnished by Zeiss through the kindness of Dr. Pulfrich and Prof. Voigt. These samples were *o* 2388 telescope crown, *o* 2546 baryt-flint and *s* 228, a new glass of remarkably high refractive index (1.9) and low dispersion. The refractive in dices of the first two varieties have been determined out to the region of absorption by H. Trommsdorff.[1]

Of each kind were a thin plate (1.5 mm.), a thick plate (5 mm.) and a wedge of sufficient angle so that only the reflection from the first surface was observed.

For a parallel-sided plate, the whole reflection from front and back surfaces sums up to be

$$R\,\frac{1 + T^2(1 - 2R)}{1 - R^2T^2}$$

[1] Inaug. Diss., Jena, 1901.

where T is the transmitting power of the plate or the ratio of the transmitted to the incident light, corrected for losses by reflection. It bears the relation

$$T = e^{-4\pi\frac{d}{\lambda}\kappa}$$

to the absorptive index κ, where d is the thickness of the plate and λ the wave-length of the light used. In the above expression for the reflection, R is the reflecting power of a single face and experimentally is the observed reflecting power of the wedge. From the reflecting powers of the wedges and the plate then, the absorptive index may be calculated. By this method the absorptive indices of the glass *s* 228 were determined and are given in the table. For the crown and flint glass, the absorption begins so abruptly and it was so difficult to obtain a source giving a sufficiently intense continuous spectrum in that region that no reliable results were obtained.

Reflecting Power of Glass.

Wave-length,	589	475	395	334	309	274	250
o 2388	4.16	4.22	4.31	4.48	4.52	4.67	
o 2546	5.37	5.63	5.86	6.58	6.94	7.48	7.83

Constants of *s* 228.

Wave-length,	589	550	500	450	400	334	274
Reflection wedge,	9.39	9.30	8.96	8.88	9.37	9.48	9.55
Reflection plate,	17.18	17.12	16.81	15.63	12.20	9.50	
T	1.00	1.00	1.00	0.954	0.604	0.031	
κ	0.00	0.00	0.00	117.10^{-8}	107.10^{-7}	615.10^{-7}	

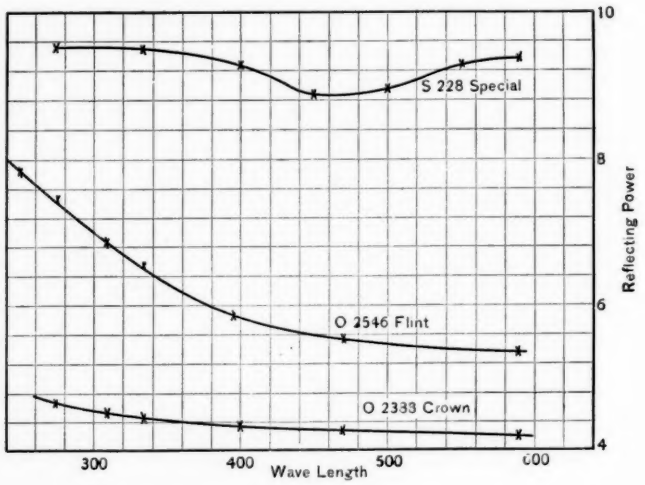

Fig 4

Reflecting Power Curves.

None of the reflecting power curves shows any abrupt increase in the reflecting power where the absorption begins. The steep part of the absorption curve for *o* 2388 lies between 298 and 304 $\mu\mu$, for *o* 2546 between 318 and 323 $\mu\mu$. There is no indication of any anomalous dispersion. From Drude's reflection formula we find that $\varkappa$ must be less than 0.001 in order not to affect the reflecting power by an observable amount. On the other hand, a plate as thin as one millimeter absorbs 99 per cent. of the incident light for $\varkappa$ as small as 0.0002. For all three kinds of glass then, $\varkappa$ lies between these limits in the ultra-violet in the region of the beginning of the absorption.

This work was done at Göttingen during the past year. In closing I wish to express my warmest thanks and appreciation for the unfailing courtesies extended by Professor Voigt, in whose laboratory the work was done.

AN ELECTRIC MICROMETER FOR LABORATORY USE.

BY PHILIP E. SHAW.

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I. DESCRIPTION OF THE INSTRUMENT.

This instrument is designed to measure any small dimension or movement. The above index shows that it has many and varied applications. It acts better (in some cases incomparably so) than the instruments at present in use for the same purposes.

Further applications will appear in Part II. of this paper.

The instrument has a unit of $\frac{1}{1000}$ mm. or micron (μ), the error being, under favorable conditions, not more than $\mu/2$. But in some readings given below the unit is only mm./200. The scale is very long, the range being from 1 μ to 40,000 μ or more.

The principle of a very sensitive instrument, the electric micrometer, has been explained in various papers.[1] It measures movements of the small amount $\frac{1}{5000000}$ mm. ($\frac{1}{5}$ $\mu\mu$). For obvious reasons this instrument would be much too delicate for ordinary laboratory measurements; but, by discarding the train of levers (or

[1] Shaw, Phil. Mag., Dec., 1900, and Phil. Mag., March, 1901. Shaw and Laws, Electrician, Feb. and March, 1901. Shaw and Laws, Electrician, March, 1902.

using only one lever in a subsidiary way), and by other simplifications, we obtain the instrument described below.

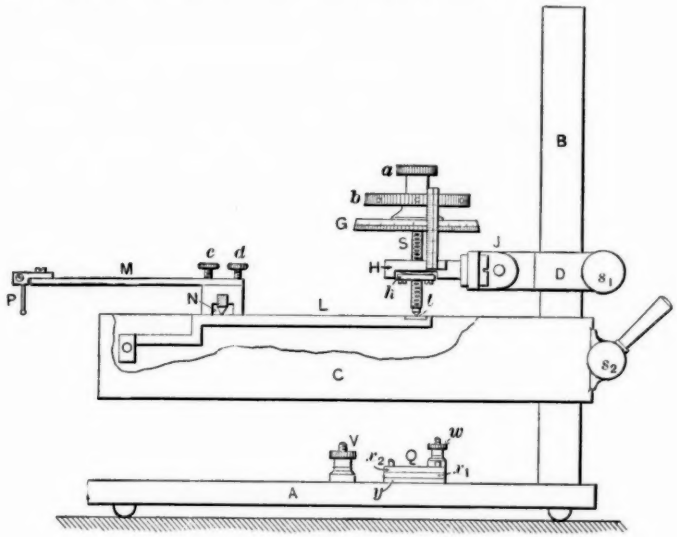

Fig. 1.

A horizontal base-plate *A*, in Fig. 1, has a vertical pillar *B* screwed to it. On this, and clamped by the clamping screw s_1, runs the sleeve *D*, carrying the measuring screw *S* and the nut *H*, there being between *H* and *D* a universal joint at *J*. By this means the screw *S* can be raised or lowered, rotated round *B*, and pointed in any desired direction; but the most common case is when, as in the figure, *S* points downward. The screw has a pitch of ½ mm. and there is a graduated disc *G*, having 500 divisions on it, so that each graduation corresponds to a movement of the screw point *t* of $\frac{1}{1000}$ mm. Above the disc *G* are two milled heads, *a* and *b*, for coarse or fine adjustment of the screw. A lock-nut *h* is shown for rendering the screw firm in the nut *H*.

For most measurements the simple mechanism so far described suffices; but in some cases it is necessary to have a lever in conjunction with the screw. If so, we require the cradle *C* which slides up and down *B* and is clamped to it by the clamping screw S_2.

On hard steel plates let into the cradle rests the fulcrum *N* of the lever *LM*. By means of the counterpoise weight *O* the lever

is kept always pressing up against the screw at t, a plate of agate as bearing surface, being let into the lever.

It is evident that, provided S points vertically down, the lever can be adjusted horizontally under S, however the latter is raised or lowered, or rotated round the pillar. There is an adjustable measuring pin P on the lever; then if the arms L and M of the lever are equal, any movement of the screw is reproduced in amount at P.

The lever is so made that the point t, the fulcrum bearing, and the end of the measuring pin P, are in one horizontal plane. By this arrangement we avoid any tangential movement or sliding at the surfaces at t and P when the lever moves. Apart from other advantages of this truly normal displacement at t and P, we insure that bad levelling or irregularity at either of the surfaces will not produce errors in the measurements.

As the object of this paper is merely to establish a new method of measuring length, nothing will be set down but very brief descriptions of methods, and only such experimental results as are required to establish the accuracy of the new method.

The processes described are meant for laboratory practice and can all be done by any student of fair experimental skill. Some of the measurements given below have been taken by students and by no means represent the best possible results.

II. A MEASURING MACHINE.

To measure thin plates and wires we use the part marked Q in Fig. 1.

Above the base-plate comes an ebonite plate y, then a steel plate x_1, and above this another steel plate x_2. x_1 and y are clamped firmly to the base-plate by screws countersunk in x_1 and passing through an ebonite insulating bush.

But x_2 is free and can be lifted off, there being ebonite pins to guide it if desired. The three plates y, x_1, x_2, are all ground true — the opposing faces of x_1 and x_2 being specially surfaced up.

Remove the cradle C and the lever, and let the measuring screw S down to nearly touch the top of x_2.

Connect up the following in series: The binding screw V, a cell, a resistance box, a telephone, the binding screw w.

Place the telephone on the head, and each time that t touches or leaves x_2 the telephone will sound.

Suppose we wish to obtain the thickness of a thin sheet of metal; we find the position of contact, (1) when the sheet is between x_1 and x_2, (2) when it is removed. The difference between (1) and (2) gives the thickness required. If the sheet is of metal and has any buckle in it, it is necessary to press x_2 down while taking the measurement.

If we wish to ascertain the diameter of a fine hair or wire we place the pieces far apart and parallel between x_1 and x_2, and proceed as before.

In order to show the possibilities and sensitiveness of the instrument a few measurements are given.

Object.	Various Measures of Thickness in Terms of μ.
Steel band.	87, 87, 86.
Silk fiber (coarse).	62, 63.
Human hair (fine).	63, 59.
Tissue paper.	58, 60.
Rice paper.	50, 52.
Mica.	30, 28, 30.
Dutch metal.	10, 11, 10.
Gold leaf (dark blue by transmitted light).	3, 5.

These measurements are taken at various places on the substance and indicate how irregular these bodies are. Reading taken at any one place agree to $\frac{1}{2}$ μ.

One special purpose for which the instrument has been used is to obtain a mica sheet of correct thickness for $\frac{1}{4}$ wave plate for the H line in the spectrum. The thickness required is about 20 μ. It is very easy to continue splitting and measuring till the required thickness is obtained.

Only thin substances are given here, though any body up to the limit of the screw (say 2 inches) can be measured with the same accuracy.

In the case of a metal block the plate x_2 can be removed, but it will be observed that for a soft or non-conducting body the upper plate is necessary.

It is a great advantage that in the electric micrometer method *pressure* is not necessary and there is no danger of crushing any of the above soft substances.

The fact that no appreciable pressure is necessary is an advantage which other measuring instruments, *e. g.*, the micrometer gauge, and Whitworth measuring machine, do not possess.

In the electric micrometer we can tighten the measuring screw in its nut, and so avoid loose working and back-lash and still retain perfect ease in taking readings. But in the other machines which depend on the *feeling* of *touch* this tightening is impossible.

The ordinary Whitworth machine measures $\frac{1}{100000}$ inch or about $\frac{1}{4000}$ mm. This can also be done with the simple electric micrometer if the divisions are divided by eye into four parts.

III. YOUNG'S MODULUS BY STRETCHING, USING THE MICROMETER.

The time-honored method of having two wires, say 10 meters long, stretching up through various floors in the laboratory building and measuring the stretch by the relative movement of a scale on one and a vernier on the other, is very crude.

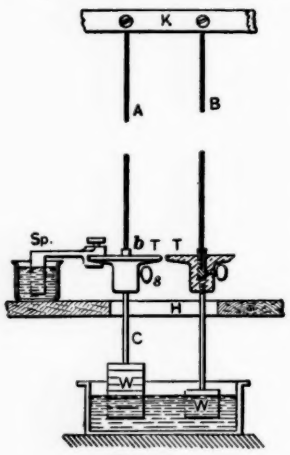

Fig. 2.

The following plan is adopted here. The wire *A*, say of steel (about 2½ meters long) is soldered firmly into the bolt *b* and this is held firm by the screw *S* to the brass platform *T*. The bottom of the platform holds the rod *C* and large weight *W*. A rod *sp* is clamped to *T* and dips into thick oil in a vessel on the table *H*, also *W* is nearly immersed in a thick oil (castor oil is used).

Place the simple electric micrometer on the table *H*. If now the micrometer screw is brought direct (or with the lever) on to the top of *T*, then as weights are added above *W*, we can measure the drop in the platform *T* and so get the stretching of the wire. The many movements to which this heavily loaded system is subject are quite obviated by the use of the two oil-baths.

The wire B, say of copper, has fittings identical with those of A and the splasher sp can be transferred to B when required. The tops of both A and B are attached rigidly to a strong cross-bar K on the same bracket fixed to the wall. If the bracket moves as a whole, or the cross-bar yields by any amount, when A is loaded, the platform of B will drop by this amount.

Hence by measuring on the platform of B when A is loaded we can discover the movement in question and deduct it in the calculations. To the usual precautions of having few kinks in the wires and starting with a large initial load must be added that the weights should be put on with their centers of gravity in a vertical line, or there will be a side movement of the platform T. It is preferable to have for C a flexible wire to minimize any tilting of T.

The wire B can be next dealt with, using A as the fixed wire. Thus we can find the modulus for two different wires on this apparatus.

It may be mentioned that the above is the last of several methods tried. It is by no means easy to obtain high accuracy, but this apparatus has proved quite satisfactory. When A was loaded and

Steel Wire. Start with 8 Kilos Load.

Down.			Up.		
Extra Weight.	Micrometer Reading.	Stretching in Terms of μ.	Extra Weight.	Micrometer Reading.	Stretching in Terms of μ.
0	950		0	951	
		>296			>299
2	654		2	652	
		>300			>298
4	354		4	354	

Repeating.

0	951		0	955	
		>297			>300
2	654		2	655	
		>299			>300
4	355		4	355	

The mean for 2 kilos $= 299\,\mu$.

the measurements taken on B the movement was $9\,\mu$ for each 2 kilos. Hence 290 represents the net extension. The modulus is then worked in the usual way.

III *a*. YOUNG'S MODULUS BY STRETCHING, USING THE OPTICAL LEVER.

There is another delicate way of measuring the stretch, viz., by the optical lever. My method of doing this is to rest two legs of the lever on the platform of *A* and the third leg on the platform of *B*. The lever carries a concave mirror which throws an image of an incandescent lamp on to a scale. The results by this means are as follows:

Steel Wire. Start with 8 Kilos Load.

Down.			Up.		
Extra Weight.	Scale Reading.	Difference in Terms of μ.	Extra Weight.	Scale Reading.	Difference in Terms of μ.
0	257, 257	$>$ 110, 110	0	255, 256	$>$ 108, 109
2	147, 147	$>$ 106, 105	2	147, 147	$>$ 106, 105
4	41, 42		4	41, 42	

Mean for 2 kilos = 107 divs.

The optical lever must be calibrated. This can be done by inserting a piece of glass of known thickness beneath the third leg of the lever. But this method of calibrating has objections, one of which is the correction for obliquity of incidence on the scale. The method I find accurate and easy is to remove the platform of *B* and to rest the third leg of the optical lever on the lever *LM* just over *P* (see Fig. 1), then by working the measuring screw *S* the optical lever is tipped and the spot runs along the scale to any desired amount.

We then find what the units on the scale are in terms of the divisions of the graduated disc *G*. Having once calibrated the optical lever we can use it subsequently with ease. On calibrating thus, the movements on the scale correspond to 298 μ and 297 μ for 2 kilos.

Thus there is very good agreement between the measurements by the simple electric micrometer and by the optical lever (see the result in section III.). But it is obvious that the former method is much more accurate than the latter while being three times as sensitive.

III*b*. An Extensometer.

It is easy to use the instrument on an engineering testing machine to determine any small movement made during a test.

IV. Young's Modulus by Flexure.

Let a metal rod be placed horizontally with its ends on two rounded knife edges, the usual care being taken to have the supports and the table specially rigid. Place the micrometer on the table bearing the supports, bring the measuring screw of the micrometer above the middle of the bar and take measurements of the position of the bar as successive kilograms are hung (through a hole in the table) from the bar.

The electric circuit is to be completed as follows: A cell, resistance box, telephone, the micrometer, the metal rod under measurement, a binding screw on one knife edge, joined to the cell.

Brass Bar.

Down.			Up.		
Extra Weight.	Micrometer Reading.	Difference in Terms of μ.	Extra Weight.	Micrometer Reading	Difference in Terms of μ.
0	248		0	247	
		>875			>874
1	1123		1	1121	
		>875			>878
2	1998		2	2001	
		>883			>880
3	2881		3	2882	
		>883			>881
4	3764		4	3764	
		>880			>876
5	4644		5	4742	
		>880			>879
6	5524		6	5524	

Mean movement for 1 kilo $= 878\mu$.

If the supports and knife-edges have been compressed by the loading, the above value will be too great. To investigate this point, place the measuring screw above the bar just over each knife-edge in succession and find if there is any depression there as the center of the bar is loaded.

In this way I found movement at the two ends to be 1 and $3\frac{1}{2}$. Taking 2 for the mean depression and deducting we obtain net movement $876\ \mu$.

The accuracy, as is shown by the numbers, is considerable. With special care the error could no doubt be reduced to 1 in 500.

It will be found very useful to have the bottom permanent weight immersed in thick oil (as in section II.) to prevent up and down vibration. The depth and breadth of the rod should be measured by the micrometer. Great accuracy is needed especially for the depth, as this enters the calculation in the third power.

V. TORSION OF A ROD.

Fig. 3 is a plan in which is shown a cylindrical rod *A* having square ends. One end is gripped firmly in jaws at *B*, the lower

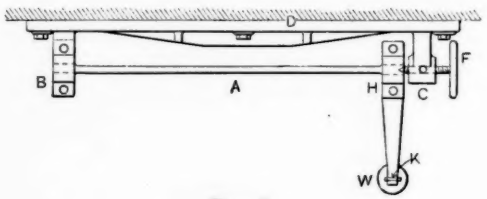

Fig. 3.

one of which is a bracket of the massive casting *D* and the upper one can be unscrewed for removal of the rod. Casting *D* is screwed to the wall.

Torsion of Steel Bar.

Down.			Up.		
Extra Load.	Micrometer Readings.	Difference in Terms of μ.	Extra Load.	Micrometer Readings.	Difference in Terms of μ.
0	70		0	75	
		>645			>640
200	715		200	715	
		>640			>640
400	1355		400	1355	
		>640			>640
600	1995		600	1995	
		>640			>640
800	2635		800	2635	
		>630			>630
1000	3265		1000	3265	

At the other end, the rod is clamped in jaws, the lower of which is the strong arm *HK* quite free from the bracket. The end of the rod is drilled to receive the coned bearing which can be screwed into it by the hand-wheel *F*. The screw is carried by the bracket *C* which is part of the main casting. Thus the left end is held and the rod will be twisted when a load is applied at *K* the end of the arm. Every part, except rod *A*, is very massive and may be trusted to be practically rigid. To ascertain the movement of *K*

when the load is applied there, place the micrometer on a table (not shown) and bring the end of the measuring screw to contact at K. As we are looking down on the apparatus K moves down when the load is applied to it. Take readings for every extra load of 200 gms. applied.

There is a slight increase in the numbers as the arm becomes oblique. This can be easily corrected for. To ascertain if the end of the rod is depressed when loads are added, I placed the measuring screw over the rod and observed a small movement, viz., $4\,\mu$ for each load added. Then as usual we have

$$\text{couple} = \frac{\pi n r^4 \theta}{2l},$$

where n = rigidity, r = radius of bar, θ = twist of bar, l = length of bar.

It is obviously easy to arrive at each of the terms and so obtain n.

VI. Thermal Expansion of a Tube.

A tube A (see Fig. 4) of steel, 1 meter long is fitted with entrance and exit tubes a and b, also with tubes for the insertion of thermometers c and d. One end of the tube is pressed against a rigid rod B which is capped with ivory, the other end of the tube is free to move.

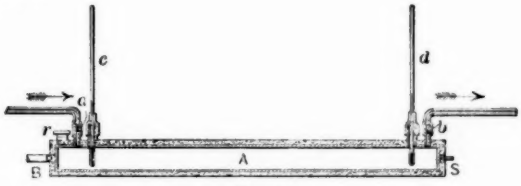

Fig. 4.

The expansion of the tube, when temperature rises, is shown by the micrometer screw S. The tube can be supported or suspended by cords, care being taken to have in either case a considerable thrust towards B.

To prevent loss of heat and quickly obtain temperature equilibrium, the tube is enclosed in a double thickness of felt, a small space being left bare at each end for the contact of B and S.

The electric circuit for measuring is arranged as in previous sections. Commence by passing steam through the tube. Plenty of steam is required for good results, so it is produced from a small boiler. After the thermometer readings have been stationary for ten minutes read the thermometers and the micrometer. Now run water from the tap, taking care not to disturb A in changing tubes at the inlet and outlet a and b, and when uniform temperature is guaranteed, take new readings with the thermometer and the micrometer.

Results for Steel Tube.

Mean Temp. of c & d.	Diff. in Temp.	Microm. Readings	Diff. in Read. in Terms of mm/200.	Coefficient of Expansions.
98.9	>92.8	338	>236	0.00001272
6.1		102		
99.4	>93.3	271	>236	0.00001265
6.1		35		

The process adopted is easy to work — thorough stirring is effected and the results are as good as can be expected with rough thermometers. The errors of temperature measurement are much greater than those of length measurement in this case. When a bar is used in place of a tube and heating is done by an electric current in a coil of wire surrounding it, we have less uniformity in temperature, but as a lecture experiment it is simple and direct.

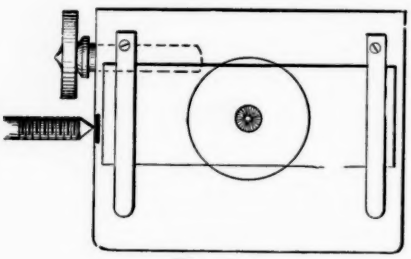

Fig. 5.

VII. MICROSCOPIC MEASUREMENT.[1]

This is one of the most simple and reliable applications of the method. Suppose we wish to measure the number of lines per centimeter in a grating. Place the grating on the stage of the microscope and focus on the lines, using a high power. Set the cross wire of the eye-piece on any line, then bring up the micrometer

[1] More detail on this point will be found in a paper by myself in The Royal Micros. Soc. Journal, December, 1902.

screw, which in this case works horizontally, to the side of the stage (see Fig. 5), and measure its position as explained in section I. Now turn the tangent screw of the stage and count the lines as they traverse the field of view. After counting 50 lines measure again the position of the side of the stage. We then obtain the difference between 50 lines.

The Lines in a Grating.

Micrometer Readings for every 50 Lines.	Difference in Terms of μ.	Micrometer Readings for every 50 Lines.	Difference in Terms of μ.
225	>454	2040	>458
679	>455	2498	>455
1134	>459	2953	>457
1593	>447	3410	

Mean 455μ for 50 lines.

Another example is the internal diameter of a capillary tube. This is shown mounted on a microscope slide on the stage in Fig. 5, so as to be viewed end-on. Then proceed as before. Two diameters at 90° to one another were taken. The bore of the tube is very elliptical in section.

Two Diameters of a Capillary Tube.

	Micrometer Readings	Difference in Terms of μ.	
1.	634 1142	>508	mean 507 μ.
	640 1147	>507	
	642 1148	>506	
2.	398 15	>383	mean 382 μ.
	381 0	>381	
	385 2	>383	

Sometimes it is very important to be able to measure a capillary tube to this degree of accuracy. Thus, in the determination of viscosity of a liquid by the rate of flow in a fine tube, the diameter of the tube enters the calculation in the fourth power.

The measurements obtained by this simple process are much more accurate than by any other available to me. In microscopes which have no tangent screw and moving stage, it is easy to arrange a parallel movement of the body and to push the body before the screw of the micrometer.

Micrometer eye-piece measurement can never be very sensitive because the micrometer scale is only magnified by the eye-piece. But the stage micrometer has the advantage of magnification by both eye-piece and objective. In the process of using Nobert's tests, two sets of readings must be taken, which involves extra trouble and error. Or if, as is commonly done, the eye-piece micrometer is calibrated by the Nobert's test the sensitiveness of the instrument is reduced, as then the measurement is taken with the eye-piece.

VIII. DIRECT MEASUREMENT OF WAVE-LENGTH.

It seemed to me that a direct measure of wave-length involving no gratings, bi-prisms, etc., would be useful and instructive. To achieve this, a change is made in the arrangement shown in Fig. 1. The arm *M* is removed. Then insert a stalk *st* (Fig. 6), carrying a convex lens in the left arm of the lever.

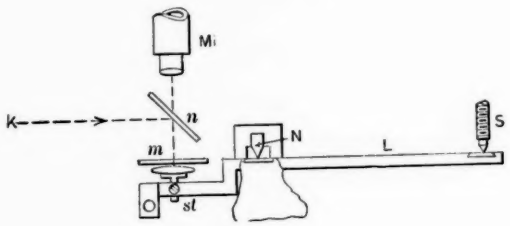

Fig. 6.

The lens is just below a microscope slide *m* and the distance between them can be made any small amount by working screw *S*. The cradle carries *m*, also another slide *n*, which can be rotated on a pin in a hole to an angle of 45° to the vertical. Allow sodium light from *K* to proceed to *n*. Then we shall be able to obtain Newton's rings between the lens and slide *m* and we can observe them by a microscope, *Mi* having a low power objective. By turning screw *S* one way, Newton's rings are produced from their

center; by reversing the screw the rings are absorbed into their center.

Then, knowing the leverage l, it is obviously quite easy to measure the wave-length λ for, suppose n microns is the movement of S required to produce 150 rings,

$$\text{the movement at the lens is } 150 \cdot \frac{\lambda}{2}$$

$$\text{the movement at the screw is } n \cdot \mu$$

and we have

$$150 \cdot \frac{\lambda}{2} \cdot l = n \cdot \mu.$$

To measure the leverage exactly is not easy. I found the following method reliable.

Place a microscope having a long horizontal traversing scale in exact line with the fulcrum edge N, then (1) traverse to a line through S and find the long arm of the lever, (2) traverse to a line through the center of the Newton's rings, hence find the short arm. In order to get the exact position of the center of the rings, place a strip of paper having a clean, straight edge, so that it appears to cut through the center of the rings. Then set the straight edge in the line of sight of the microscope and proceed to take the reading of the straight edge.

There are two special difficulties to be noticed in working by this method. (1) The stalk carrying the lens can be adjusted up and down, but if precautions are not taken in this adjustment, the center of curvature of the lens will not move normal to the surface of the slide m. If the lens is too high, then, as the lens rises, the center of curvature moves outwards — if too low it moves inwards and in either case the results are variable. But by trial an intermediate position is found when the readings are steady. In the table below a small progressive increase is seen, showing that exactly the best position was not used in this case.

(2) The milled head of the measuring screw must be turned very steadily so that the rings may be accurately counted. A long pin is thrust into the capstan holes in b (see Fig. 1). If this long

leverage is used and the hand is steadied on a rest, the process is quite easy.

Newton's Rings. Readings for Every 50 Rings.

Down.		Up.	
Microm. Reading in Terms of μ.	Difference for 150 Rings.	Microm. Reading in Terms of μ.	Difference for 150 Rings
365.2		365.2	
324.2		324.0	
	> 123.9		> 123.5
283.0		283.0	
	> 124.4		> 124.7
241.3		241.8	
	> 125.8		> 125.8
199.8		199.3	
157.2		157.2	

Mean for 150 rings 124.7 μ.

Measurement of Leverage.

	Short Arm.		Long Arm.	
Right end reading.	7.57	7.57	13.72	13.72
Left end reading.	3.36	3.37	1.75	1.74
Difference.	4.21	4.20	11.97	11.98
	Mean 4.205		Mean 11.975	

Using the expression

$$\lambda = \frac{2 \cdot n \cdot \mu}{150 \cdot l},$$

we find

$$\lambda = 58 \cdot 61 \times 10^{-6} \text{ cm.}$$

This is fairly near the true value ($58 \cdot 95 \times 10^{-6}$) for the mean of the D lines.

IX. PRECAUTIONS.

In the paper on the electric micrometer[1] directions are given whereby the fine measurements there mentioned are rendered possible. The present instrument, though so much less sensitive, yet is delicate and requires care in the following points:

1. The measuring screw point and the surface opposed to it must be smooth and should be kept clean by the finest emery paper. A little oil, dirt, or varnish renders the readings uncertain.

[1] *Loc. cit.*

2. Firm tables attached to the walls or on the firm ground floor are desirable. Vibrations from outside causes are thus avoided. Moreover, it will be very often found impossible to lean on the table without effecting the readings considerably.

3. The lock-nut of the measuring screw must be adjusted nicely so that the screw turns firmly in its nut or back-lash may occur.

4. The circuit of, a cell, telephone, resistance box, measuring-screw and the contacts is easy to arrange. The resistance is put as high as will give distinct but faint sounds. Thus I commonly use a current of only $\frac{1}{5000}$ ampère. Such a current will not be likely to heat the contacts much, or cause them to expand appreciably.

5. The best form of telephone is a "watch-receiver" provided with an attachment which fits on the head and leaves both hands free.

6. Damping should be attended to, notably in Young's modulus by stretching, in which case it is essential.

7. The measuring screw must always be set perpendicular to the surface on which it works.

X. Summary.

All the processes described have been actually made to work. The scantiest experimental results are given; if time allowed, far better results could be obtained, but the object of this paper is to establish the method and not to obtain special results.

The instruments which could be used for taking such measurements as are mentioned in this paper and which have a great range of application are (1) the microscope, (2) the optical lever, and (3) the simple electric micrometer.

A good instrument should have: (1) delicacy; (2) accuracy; (3) length of scale. Let us compare the rival instruments in these respects.

Microscope.—A micrometer eye-piece has, say fifty lines, ruled to a mm.; to go further than this is not much use unless the light is very good. Thus the instrument is not very sensitive. If a micrometer screw eye-piece is used we obtain no more delicacy (although some instruments pretend to give $\frac{1}{500}$ mm.).

If the eye-piece has 500 divisions to 1 mm. the cross-wire will have to be moved 10 or 12 divisions to make any movement perceptible to the eye. This means that the unit is $\frac{12}{500} = \frac{1}{40}$ q. p. as before.

The unaided eye can resolve 4 lines to 1 mm. at 10 inches distance; with the help of an eye-piece, of magnifying power 10, the eye resolves 40 lines to 1 mm. When an eye-piece micrometer is used, this is all that is obtainable, the unit of measurement being $\frac{1}{40}$ mm. about. If however a stage micrometer of some sort is adopted the full resolving power of the microscope is used on the scale of measurement and as a limit 4,000 lines to the mm. can be obtained for the best possible microscope. But as pointed out in Section VII., these stage micrometer measurements are indirect and hence liable to error. The simple electric micrometer gives the same fineness of measurement but has the many advantages of being direct-acting.

The range of the microscope is small except by a step-by-step process which is objectionable. Hence the microscope as generally used, is accurate, but not delicate or long in scale.

Optical Lever.—Using no levers, but merely a three-legged stand carrying a mirror, we can easily get as unit $\frac{1}{500}$ mm., or possibly $\frac{1}{1000}$ mm. But there are two grave objections, (1) we have to correct for changing obliquity of the spot on the scale, (2) the calibration of the instrument is difficult to perform accurately and itself involves errors.

The scale is not long; it is necessarily limited, except by the awkward step-by-step process.

Hence the optical lever may be said to be delicate, but not accurate or long in scale.

Simple Electric Micrometer.—As to delicacy, $\frac{1}{1000}$ mm. can be obtained easily. Dividing by eye $\frac{1}{10000}$ mm. is obtainable. This is done in section VIII. The accuracy is great, successive readings being in excellent agreement. The sources of inaccuracy in the optical lever do not here exist. As to length of scale, the instrument works from 1 μ to 40,000 μ or more if desired.

Besides having delicacy, accuracy, and great length of scale, the instrument is adaptable to any sort of experiment. There is no

kind of small measurement, as far as I know, which the apparatus cannot make.

The arrangement shown in Fig. 1 need not be costly, and the accessories, cell, telephone and resistance-box, are in every laboratory.

A unit of much less than $\frac{1}{1000}$ mm., say $\frac{1}{10000}$ mm., could be obtained by having a larger graduated disc or by using levers, but if any greater accuracy is required it would be necessary to avoid vibrations by hanging the apparatus on rubber suspensions, and thus the present simplicity would be lost.

There is one simple improvement on the instrument as described above. A pulley groove can be worked on the large milled head (Fig. 1) and the screw rotated by a pulley string and outside hand pulley. The hand if used directly on the screw may be found to be too unsteady for the finest work.

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THE DAMPED BALLISTIC GALVANOMETER.

BY O. M. STEWART.

THE theory of the ballistic galvanometer as usually presented assumes either that there is no damping or that it is small. Consequently there is a general impression that a well-damped ballistic galvanometer cannot be used to measure or compare quantities of electricity. Yet it has been shown experimentally that such a galvanometer can be used. When used in determinations of magnetic induction[1] no appreciable errors have been detected. However this has not exempted such use from some criticism.[2]

Theoretically, damped ballistic galvanometers may be divided into two classes: one where the motion is vibratory or periodic, the other where the motion is aperiodic. As the theory of the latter does not seem to be available it has been thought advisable to present here a brief discussion of the subject.

The general equation of the motion of a vibrating system is

$$K\frac{d^2\theta}{dt^2} + \alpha\frac{d\theta}{dt} + \beta\theta = 0 \qquad (1)$$

where K is the moment of inertia, $\alpha\, d\theta/dt$ the moment of the retarding force due to the damping and $\beta\theta$ the moment of the force of restitution. If the galvanometer is of the D'Arsonval type β is the moment of torsion of the suspension. If of the other or Kelvin type

$$\beta = MH$$

where M is the magnetic moment of the needle and H is the horizontal component of the earth's field.

[1] Sheldon and Cocks, Electrical World, 31, p. 735, 1898.

[2] See Discussion in Proc. Am. Inst. E. E., Vol. 17, pp. 15 and 18, 1900.

The solution of equation (1) is

$$\theta = Ae^{\mu_1 t} + Bc^{\mu_2 t}$$

where

$$\mu_1 = -\frac{a}{2K} + \lambda$$

and

$$\mu_2 = -\frac{a}{2K} - \lambda$$

and

$$\lambda = \sqrt{\frac{a^2}{4K^2} - \frac{\beta}{K}}.$$

If an initial velocity ω_0 be imparted to the suspended system the initial conditions are when $t = 0$,

$$\theta = 0, \quad \frac{d\theta}{dt} = \omega_0.$$

If this initial velocity is given by the discharge of a quantity Q through a D'Arsonval galvanometer

$$\omega_0 = \frac{ANHQ}{10K},$$

where A is the mean area of the coil, N the number of turns, H the intensity of the magnetic field and K the moment of inertia.

This may be written

$$\omega_0 = cQ.$$

For the Kelvin type

$$\omega_0 = \frac{MGQ}{10K} = cQ$$

where G is the "coil constant" of the galvanometer and M and K have the same meaning as before.

Applying these initial conditions the solution of equation (1) becomes

$$\theta = \frac{cQ}{2\lambda}(e^{\lambda t} - e^{-\lambda t})e^{-\frac{a}{2K}t}. \qquad (2)$$

If λ is imaginary, that is if

$$\frac{a^2}{4K^2} < \frac{\beta}{K}$$

the motion is periodic and equation (2) becomes

$$\theta = \frac{cQ}{\sqrt{\frac{\beta}{K} - \frac{a^2}{4K^2}}} e^{-\frac{a}{2K}t} \sin t \sqrt{\frac{\beta}{K} - \frac{a^2}{4K^2}}. \tag{3}$$

This is the equation usually given. However, if, the damping is so great that λ is real the motion is aperiodic and equation (2) gives the value of θ.

From equation (2) it is seen that θ at first increases, reaches a maximum and returns to the zero position (theoretically) after an infinite time.

If the time it takes θ to reach its maximum value in equation (2), is independent of Q, that is, if this time is a constant, the maximum value of θ, or the throw, is directly proportional to Q. This can be shown as follows:

Let t_1 be the time when $\frac{d\theta}{dt} = 0$, that is when θ is a maximum.

Then from (2)

$$\frac{d\theta}{dt} = 0 = \frac{cQ}{2\lambda} e^{-\frac{a}{2K}t_1} \left[\lambda(e^{\lambda t_1} + e^{-\lambda t_1}) - \frac{a}{2K}(e^{\lambda t_1} - e^{-\lambda t_1}) \right]$$

or

$$\lambda(e^{\lambda t_1} + e^{-\lambda t_1}) = \frac{a}{2K}(e^{\lambda t_1} - e^{-\lambda t_1}).$$

This may be written

$$e^{2\lambda t_1} = \frac{\frac{a}{2K} + \lambda}{\frac{a}{2K} - \lambda}. \tag{4}$$

Hence t_1 is independent of Q, and is a constant. Equation (2) may be written

$$\theta_1 = \frac{cQ}{2\lambda}(e^{\lambda t_1} - e^{-\lambda t_1}) e^{-\frac{a}{2K}t_1} \tag{5}$$

where θ_1 is the throw and t_1 the time of the throw.

Since all the quantities in the right member of (5) are constants with the exception of Q, (5) may be written

$$\theta_1 = kQ.$$

Or the throw in a galvanometer whose motion is aperiodic is directly proportional to the quantity and therefore conforms to the general rule of undamped ballistic galvanometers.

In the same way it can be shown that the time it takes θ to reach its maximum value in (3) where the motion is periodic is also a constant and the throw is directly proportional to Q. Hence, no matter how large the damping is, the throw is directly proportional to the quantity of electricity suddenly discharged through it.

The assumption has been made that the damping force is directly proportional to the velocity. If the damping is due to induced currents, no matter where, this is strictly true. In the case of air damping it is approximately true. It is also assumed that the force of restitution is proportional to the angular deflection. This for the movable needle type is only true for small deflections, but for the D'Arsonval galvanometer it is not so restricted. If the magnetic field of the D'Arsvonal galvanometer is not uniform, it will affect this general law only as it may affect the damping factor α.

The quantity

$$\frac{\alpha^2}{4K} - \frac{\beta}{K} = \lambda^2$$

is negative for periodic and positive for aperiodic vibrations. When the damping is small or negligible, this quantity is not large as the period of vibration is $\frac{2\pi}{\sqrt{-\lambda^2}}$. As the damping is increased λ^2 increases, passes through zero and becomes positive. Hence for a considerable range of damping λ^2 is small.

If the exponential functions in equation (5) containing λ be expanded the equation becomes

$$\theta_1 = cQt_1\left(1 + \frac{\lambda^2 t_1^2}{3!} + \frac{\lambda^4 t_1^4}{5!} + \cdots\right) e^{-\frac{\alpha}{2K}t_1}. \qquad (6)$$

From this it is seen that excepting the effect the change has on t_1, the value of the throw is approximately independent of λ^2. From equation (4) it may be shown that changes in λ have but littel effect on t_1 provided $\frac{\alpha}{2K}$ remains constant. Since $\lambda^2 = \sqrt{\frac{\alpha^2}{4K^2} - \frac{\beta}{K}}$

the changes in λ are confined to changes in β. Hence in an aperiodic galvanometer the throw is approximately independent of β. Or if the galvanometer is one where the magnet moves, the throw is approximately independent of changes in the earth's field. This was verified with an Edelmann galvanometer whose bell-shaped magnet is well damped by a copper damper. On reversing the controlling magnet so that the steady deflection for a certain current was about *four times* as great as before, it was found that the throw due to a discharge was increased only about 15 per cent. In the case of an aperiodic D'Arsonval galvanometer the throw is approximately independent of the moment of torsion of the suspension strips since β is here equal to the moment of torsion. This is of some practical importance, as it often happens that in low resistance galvanometers of this type the suspension strips have as great and sometimes greater resistance than the coil itself. When this galvanometer is intended for ballistic work with an earth inductor or in electromagnetic experiments where the damping is necessarily great, the galvanometer being used on closed circuit, a considerable gain may be had by not making the strips so small.

These facts may also be explained by saying that in a well-damped galvanometer the principal agent in overcoming the velocity imparted to the needle and thus limiting the throw is not the earth's field or the torsion of the suspension but the damping.

In the special case where $\lambda^2 = 0$, one has from (2)

$$\theta = cQte^{-\frac{a}{2K}t}. \tag{7}$$

By differentiating this with respect to t and remembering that $\frac{d\theta}{dt} = 0$ when $t = t_1$, one obtains

$$1 - \frac{at_1}{2K} = 0,$$

or

$$t_1 = \frac{2K}{a}.$$

Or

$$\theta_1 = \frac{2cQK}{a} e^{-1}.$$

Here θ_1 is independent of changes in the earth's field or suspension except as those changes affect the condition that λ is zero. That is the rate of change of θ_1 with respect to these changes is at this point zero.

Since
$$\frac{a^2}{4K^2} - \frac{\beta}{K} = \lambda^2 = 0$$

$$t_1^2 = \frac{K}{\beta}. \tag{8}$$

If there is no damping the period of vibration is

$$T = 2\pi \sqrt{\frac{K}{\beta}}. \tag{9}$$

From (8) and (9)
$$\frac{T}{t_1} = 2\pi. \tag{10}$$

Or the undamped period is 2π times the time of throw when the damping is just sufficient to make the needle aperiodic. Since the time of the throw when undamped is one fourth of the period the effect of this damping is to shorten the time of the throw by the factor $2/\pi$, that is to about two thirds of its undamped value.

In this special case the effect of the damping on the sensibility of the instrument is readily obtained. The value of the undamped throw or the maximum value of θ may be shown from (3) to be

$$\theta_1 = \frac{cQ}{2\pi} T \tag{11}$$

where T has the value given in (9).

Since when damped
$$\theta_1 = cQt_1 e^{-1}$$

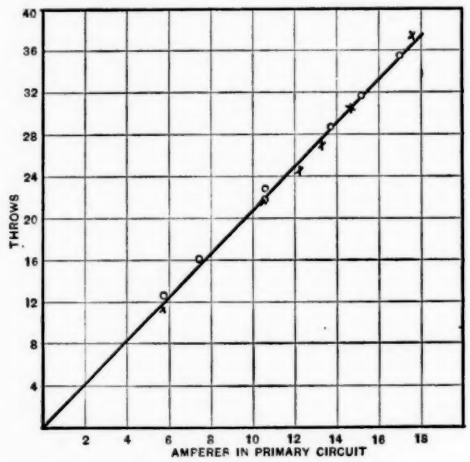
one has as the ratio of the damped throw to the undamped

$$\frac{2\pi t_1}{eT} = \frac{1}{e}$$

from (10).

Or the throw when the damping has been increased so that the motion is just aperiodic, that is does not swing past its zero position on the return, is about 37 per cent. of the undamped throw.

In Fig. 1 is given a curve showing the relation between the throws of a Weston milliammeter and the quantity discharged through it. The milliammeter was connected to the secondary of a



The Ballistic Calibration of a Weston Ammeter.

Fig. 1.

solenoid wound on a wooden core. The throw was obtained on making the current in the primary circuit. Since the quantity discharged through the secondary was proportional to the current in the primary circuit, currents have been plotted instead of quantities. It is seen from the curve that the throws are proportional to quantity. Since the throws of the index of the ammeter are very quick they are rather hard to read. The accuracy of reading may be judged from the curve, as two sets of observations taken by different persons are given. These persons had had but little previous practice in reading the throws of such an instrument.

In all the determinations of magnetization curves and hysteresis loops by the ballistic method a great deal of time may be saved by using as the ballistic galvanometer an ammeter without its shunt,[1] or a voltmeter without its series resistance.

[1] The Weston Type E, a portable instrument, and nearly all switch-board types are suitable, as they have external shunts.

There were two drawbacks to the use of the damped galvanometer or ammeter as a ballistic galvanometer:—first, its decreased sensibility, and second, that one cannot determine its constant from its period and figure of merit. In most work this method is not used, it being more convenient to use a solenoid. The decrease in the sensibility is in most cases not a serious drawback, for it rarely happens that this must be very great. Damped D'Arsonval galvanometers that are quite satisfactory for nearly all ballistic work are not only available, but far more convenient than the undamped galvanometers.

UNIVERSITY OF MISSOURI,
December 20, 1902.

REFLECTION FROM WALLS OF PHOTOMETER ROOM.

BY F. J. ROGERS.

AN experiment suitable for junior students which will demonstrate satisfactorily that the intensity of illumination varies inversely as the square of the distance from the source is not easy to arrange. The difficulty in varying the intensity of a source in a known ratio, the irregular variation of both sources, reflection from walls of photometer room, and lack of sensitiveness of the eye for small differences of illumination all conspire in producing results which are very unsatisfactory.

I had arranged an experiment of this sort in which the variable source consisted of one or more four-candle-power incandescent lamps. Instead of using a photometer room the lamps and photometer were enclosed in long photometer boxes, the head of the observer being covered with a black cloth to keep out stray light. The object of the experiment was to determine the relation between the intensity of illumination on one surface of the photometer screen and the distance to the source producing the illumination. The results obtained were worked up by plotting the logarithms of intensity and of distance. The slope of the resulting curve (which is a straight line) gives the power of the distance according to which the intensity of the illumination varies. The results obtained by my students gave, instead of the inverse square, inversely as the 1.8 to 1.9 power, or in symbols $I = kD^{-1.85}$. The fact that this power is less than 2 is readily explained by the irregular reflection which takes place from the walls of the photometer box. I was surprised not so much at the amount of reflection as at the deviation produced from the theoretical inverse square law, and determined to give the matter a further test in a photometer room.

Several series of observations were made in which the variable source consisted of one or more of eight four-candle-power lamps.

A Lummer-Brodhun photometer was placed at a *fixed* distance from the 4-c.-p. lamps while a 16-c.-p. lamp on the other side of the photometer was *moved* until the two sides of the photometer screen were equally illuminated. This was repeated for one, two, three, four, six and eight of the 4-c.-p. lamps. The 4-c.-p. lamps were approximately equal in illuminating power, but observations were so taken as to eliminate from the results the effect of the unavoidable inequalities in the different lamps. For example, when one lamp was used observations were made with each of the 4-c.-p. lamps separately and the mean taken; when six lamps were used observations were made successively with — all but Nos. 1 and 2, all but 3 and 4, all but 5 and 6, and all but 7 and 8.

Assuming the relation $I = kD^n$, the value of n was determined by the method of least squares from the observations given below and found to be — 1.92. Several series of preliminary observations gave for n values very nearly equal to the one just given. In order to be certain that the deviation from the inverse square relation was due to reflection from the wall of the photometer room the latter was draped — wherever any perceptible reflection to the photometer was probable — with black canton flannel plaited so as to reflect the least possible light.

Another series of observations were taken under exactly the same conditions. When n was computed from this series of observations it proved to be almost precisely equal — 2 as demanded by the inverse square law.

No. of 4-c.-p. Lamps at fixed Distance from Photometer.	Distance from Photometer to Movable Source.	
	With Drapery on Wall Bet. Photometer and Mov. Source.	Drapery "Pinned Up."
1 Lamp.	206.0 cm.	220.2 cm.
2	144.6	155.6
3	117.8	125.9
4	101.7	107.4
6	84.0	86.7
8	72.7	75.4

Movable source, a 16-c.-p. lamp with cylindrical frosted bulb.
Equations derived from above data:

I. Wall Draped. II. No Drape.

$\text{Log } I = -2.00 \text{ Log } D + 4.6$; $\text{Log } I = -1.92 \text{ Log } D + 4.5$.

The table on page 173 gives the data from which was computed the value of n given above. Each "distance" given in the table is the mean of eight or sixteen different observations so taken as to eliminate the effect of inequalities among the 4-c.-p. lamps.

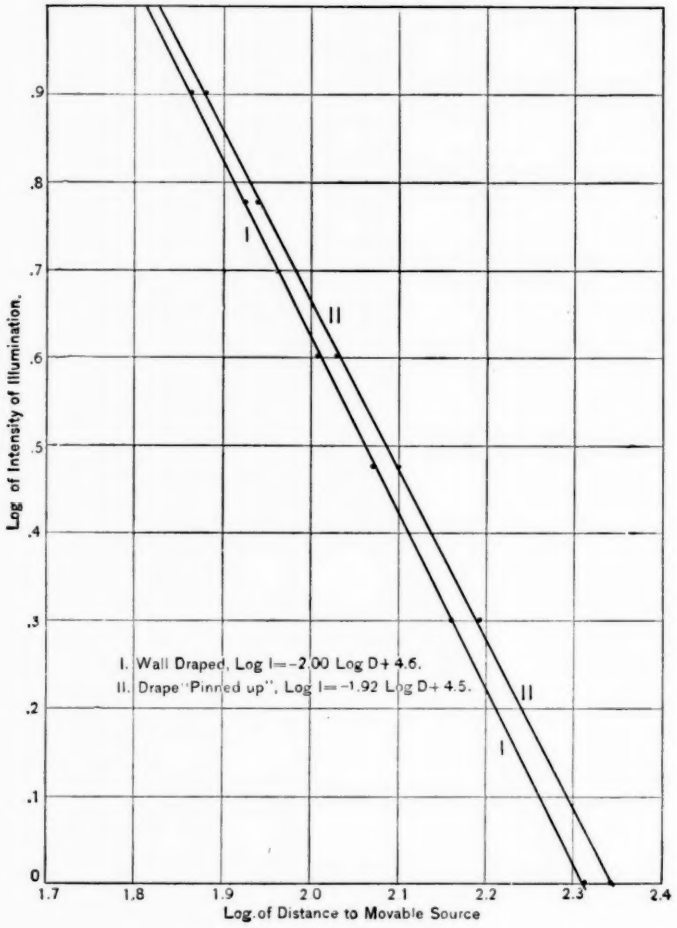


Fig. 1.

In the plot, Fig. 1, the logarithms of the data given in the table are plotted. The shifting of the line produced by removing the drapery from the wall illustrates the reflection from the walls of the photometer room. The change in slope illustrates the change in

the law which connects intensity of illumination with distance from source.

It is a little surprising that the use of drapery on the black wall of a photometer room should reduce the light reaching one side[1] of the photometer from 7% to 15% and at the same time change the relation of intensity to distance from the inverse 1.92ths power to the usual inverse 2d power. In the present case, however, the conditions were rather favorable for reflection, as the photometer bar was less than 30 cm. from the wall and the latter was painted with asphalt paint which did not give a "dead" black surface.

STANFORD UNIVERSITY, May, 1902.

[1] The drapery was placed so as to prevent reflected light from falling on one side of the photometer screen, the other side of the photometer was entirely unaffected by it.

WASHINGTON MEETING OF THE AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE.

THE experiment of holding a winter meeting of the American Association during Convocation Week, tried this year for the first time, proved eminently successful, the attendance being larger than at any meeting in recent years. The favorable location of Washington and the many attractions which this city offers to scientific visitors doubtless contributed much to the success of the meeting; yet it can hardly be doubted that the change in the time of the meeting was also a factor of importance.

Following the custom established at New York in 1900, a meeting of the American Physical Society was held in connection with the meeting of Section B of the American Association. On Wednesday, December 31, and Thursday, January 1, a joint session of the two societies was held. On the former day the program was made up of Physical Society papers only.

While the attendance in Section B, and in the joint session with the American Physical Society, was unusually large, the program was not so crowded as it has often been in the past. In addition to the address of the retiring chairman of Section B, Prof. W. S. Franklin, on "Popular Science," twenty-six papers were read before Section B. In the joint session with the Physical Society, nineteen papers were presented, making a total of forty-five. The hurried feeling that has often been felt at the meetings of the association, caused by the necessity of completing a too extended program, was less marked than usual, and a more extended discussion of the papers presented naturally resulted. It is to be hoped that this feature of the meetings will become even more prominent.

The following is a list of the papers presented to Section B:[1]

The Semidiurnal Periods in the Earth's Atmosphere. Frank H. Bigelow.

The Construction of a Sensitive Galvanometer. C. G. Abbott.

The Conditions Governing the Coherence of Metals when there is an Electrical Discharge between Them. Carl Kinsley.

A Determination of the Frequency of Alternating Currents by the Automatic Adjustment of the Circuit to Resonance. Carl Kinsley.

[1] A list of Physical Society papers will be found on p. 173.

On Methods of Measuring Radiant Efficiency. E. L. Nichols and W. W. Coblentz.

The Infra-red Spectrum of the Mercury Arc. W. W. Coblentz and W. C. Geer.

Derivation of Equation of Decaying Sound in a Room, and Definition of Open Window Equivalent of Absorbing Power. W. S. Franklin.

On the Velocity of Light as Affected by Motion Through the Ether. Edward W. Morley and Dayton C. Miller.

Some Measures of the Speed of Photographic Shutters. Edward W. Morley and Dayton C. Miller.

On the Distribution of Pressure Around Spheres Moving with Constant Velocity in a Viscous Fluid. S. R. Cook.

A Portable Apparatus for the Measurement of Sound. A. G. Webster.

The Damped Ballistic Galvanometer. O. M. Stewart.

On the Electrical Conductivity of Solutions in Amyl Amine. Louis Kahlenberg.

On the Thermal Conductivity of Glass. H. W. Springsteen.

Some Relations Between Science and the Patent Office. Charles K. Wead.

Why the E.M.F. of the Daniell Cell Changes when the Densities of the Solutions Change. Henry S. Carhart.

Absolute Measurement of the E.M.F. of the Cadmium Cell. Henry S. Carhart and Karl E. Guthe.

The Characteristic Absorption Curves of the Permanganates. B. E. Moore.

The Magnetic Rotary Dispersion of Solutions of Anomalous Dispersive Substances. F. J. Bates.

Atmospheric Circulation Near the Equator. A. L. Rotch.

The Anomalous Dispersion and Selective Absorption of Fuchsin. Wm. B. Cartmel.

Sulphur Dioxide and the Binary Vapor Engines. R. H. Thurston.

The Coefficient of Expansion of Some Alloys of Nickel and Cast Iron. Theo. M. Focke.

A New Apparatus for Demonstrating Wave Motion. Fred. J. Hillig.

Demonstration of a Portable High Tension Coil. G. Lenox Curtis.

The next meeting of the Association will be held at St. Louis during Convocation week, 1903 — *i. e.*, during the week beginning Monday, December 28, 1903. No meeting will be held during the coming summer. The omission of the summer meeting in 1903 is not to be regarded, however, as indicating the permanent policy of the association. Special reasons were advanced in the Council for omitting the summer meeting for this year; but a strong sentiment exists in the Association in favor of meetings during the summer, so that it seems not unlikely that in subsequent years two meetings may be held.

The officers of Section B for the coming year are E. H. Hall, chairman, and D. C. Miller, secretary. The Sectional Committee consists of Messrs. E. H. Hall, D. C. Miller, E. F. Nichols, G. F. Hull, A. G. Webster, D. B. Brace, Ernest Merritt, and C. E. Mendenhall.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE EIGHTEENTH MEETING.[1]

THE annual meeting of the American Physical Society was held at the Columbian University, Washington, D. C., on Wednesday, December 31, 1902, and Thursday, January 1, 1903, in conjunction with the meeting of Section B of the American Association for the Advancement of Science.

Vice-President Webster presided.

The following officers were elected for the year 1903:

President, Arthur G. Webster; *Vice-President*, Elihu Thomson; *Secretary*, Ernest Merritt; *Treasurer*, William Hallock; *Members of the Council*, W. F. Magie and E. H. Hall.

The following papers were presented:

Results of Comparisons of Magnetic Instruments. L. A. BAUER.

The Pressure Due to Radiation (Final Results). E. F. NICHOLS and G. F. HULL.

Is there a Southerly Deviation of Falling Bodies? EDWIN H. HALL.

Elasticity of Copper and Steel at — 186° C. J. R. BENTON.

Thermodynamic Formulæ for Isotropic Solids subject to Tension. J. R. BENTON.

Experiments in Connection with Friction Between Solids and Liquids. J. R. BENTON.

The Heat of Vaporization of Oxygen and Nitrogen. J. S. SHEARER.

The Magnetic and Electric Deviation of the Easily Absorbed Rays from Radium. E. RUTHERFORD.

A Penetrating Radiation from the Earth's Surface. E. RUTHERFORD and H. L. COOK.

Induced Radioactivity Excited at the Foot of Water Falls. J. C. MCLENNAN.

[1] The minutes of all meetings of the American Physical Society and abstracts of papers presented to the Society will hereafter be published in the PHYSICAL REVIEW. The publication of the *Bulletin of the American Physical Society* has been discontinued.

Some Experiments on the Electrical Conductivity of Air. J. C. McLENNAN and E. F. BURTON.

Radioactivity of Freshly Fallen Snow. S. J. ALLEN.

On the Double Refraction of Dielectrics in a Magnetic Field in a Direction at Right Angles to the Lines of Force. D. B. BRACE.

The Viscosity of Liquids at Low Rates of Shear. A. WILMER DUFF.

On the Mechanical Efficiency of Musical Instruments in Producing Sound. A. G. WEBSTER.

Experiments Concerning very brief Electrical Contacts. HERSCHEL C. PARKER.

A Comprehensive Boyle's Law Apparatus. W. J. HUMPHREYS.

Lecture Room Method of Analyzing Irregular Currents. W. J. HUMPHREYS.

The Critical Current Density and Drop of Protential at the Cathode in Vacuum Tubes. C. A. SKINNER.

Adjourned.

ERNEST MERRITT,
Secretary.

AN EXPERIMENT RELATING TO THE APPLICATION OF LAGRANGE'S EQUATIONS OF MOTION TO ELECTRIC CURRENTS.[1]

BY WILLIAM S. DAY.

THE experiment described was analogous to one mentioned by Maxwell in his Treatise on Electricity and Magnetism, Section 574, Volume II. For the purpose of applying Lagrange's equations of motion to the phenomena of electric currents, Maxwell considered that the state of a system of electric currents could be given by two kinds of generalized coördinates, the mechanical coördinates which indicate the positions of the wires or conductors carrying the currents, and the electrical coördinates whose velocities represent the currents flowing. Following out the analogy of a purely mechanical system, the kinetic energy of an electrical system would in general be a homogeneous quadratic function of the velocities of the coördinates. For one linear circuit alone, having one degree of mechanical freedom, this would be of the form,

$$T = \tfrac{1}{2}L\dot{x}^2 + K\dot{x}\dot{y} + \tfrac{1}{2}L\dot{y}^2$$

in which T = the kinetic energy, $\dot{x}$ = the velocity of the mechanical

[1] Abstract of a paper presented at the meeting of the Physical Society held on April 21, 1902.

coördinate, $\dot{}$ = the current, and I, K and L are coefficients which may be constants, or functions of x, but as Maxwell shows, not of y. I of course corresponds to a mass or a moment of inertia, and L is the self-induction of the circuit. Maxwell's experiment referred to above was for the purpose of finding out whether or not any part of the kinetic energy of the circuit should be expressed by the term $K\dot{x}\dot{y}$. In his experiment he takes as his mechanical coördinate one whose variation expresses motion of the wire in the direction of its length. If such a term existed, a wire would experience a sudden impulse in the direction of its length when a current was suddenly started, or suddenly stopped in the wire. Maxwell states that no evidence of the existence of such a term has ever been discovered. So far, however, as Lagrange's equations of motion themselves are concerned, and apart from any hypothesis as to the nature of an electric current, a different mechanical coördinate might have been considered, namely, one whose variation expresses rotation of the wire around its axis. If the term $K\dot{x}\dot{y}$ had an existence, this being the meaning attached to the mechanical coördinate, the wire would experience an impulsive torque when a current was suddenly started or stopped in it. The present experiment was made for the purpose of discovering such an effect if it existed, or at least of fixing as small a maximum limit to its value as possible. For this purpose a straight aluminum wire 30 cm. long and 0.25 cm. in diameter was suspended by a quartz thread so that while it was free to rotate about its axis a current of 10 ampères could be passed through it at will by means of mercury cups, in either direction. The actual moment of inertia of the system was about 0.07 C.G.S. units, and the free period of vibration around the axis about 2 seconds. After reducing all disturbing forces to a minimum, no certain indication of such an effect could be detected. If the coefficient K supposed to be a constant as the simplest hypothesis, and referred to unit length of the wire, had had a value as great as about 0.00002, measured in C.G.S. electromagnetic units, it could have been detected. If K had this value it would mean that if a current of 10 ampères in a wire were made to decrease to zero at a uniform rate in exactly one second there would be during this second a constant torque acting on the wire on each centimeter of its length equal to a force of 0.00002 dyne with a lever arm of 1 centimeter.

This coefficient could also be tested for by observing the electromotive force, if any, produced when a rapidly rotating wire is suddenly brought to rest.

A NEW METHOD OF INTEGRATING ONE OF THE DIFFERENTIAL EQUATIONS OF THE THEORY OF HEAT DIFFUSION.[1]

BY R. S. WOODWARD.

MANY problems in the theory of heat diffusion require integration of the equation

$$\frac{\partial v}{\partial t} = a^2 \frac{\partial^2 v}{\partial x^2} \tag{1}$$

subject to given initial and boundary conditions. In this equation a^2 is a constant (diffusivity), t is time after some initial epoch, x is distance and v is temperature or the product of temperature by x.

To find an integral of (1), assume

$$2az\sqrt{t} = q \pm x, \tag{2}$$

where q is an arbitrary constant distance and z is a new variable. Thus v may be regarded as a function of z, and it follows that

$$\frac{\partial v}{\partial t} = \frac{\partial v}{\partial z}\frac{\partial z}{\partial t} = -\frac{\partial v}{\partial z}\frac{z}{2t},$$

$$\frac{\partial v}{\partial x} = \frac{\partial v}{\partial z}\frac{\partial z}{\partial x} = \pm\frac{\partial v}{\partial z}\cdot\frac{1}{2a\sqrt{t}},$$

$$\frac{\partial^2 v}{\partial x^2} = \frac{\partial^2 v}{\partial z^2}\cdot\frac{1}{4a^2 t};$$

whence

$$\frac{\partial^2 v}{\partial z^2} = -2z\frac{\partial v}{\partial z}.$$

The integral of this is

$$\log\left(\frac{\partial v}{\partial z}\right) = -z^2 + \log C$$

or

$$\frac{\partial v}{\partial z} = Ce^{-z^2},$$

whence

$$v = C\int_{z_1}^{z_2} e^{-z^2}dz. \tag{3}$$

Referring to (2) it is seen that the limits z_1 and z_2 may be replaced by

$$\frac{q+x}{2a\sqrt{t}} \quad \text{and} \quad \frac{q-x}{2a\sqrt{t}}$$

[1] Abstract of a paper presented at the meeting of the Physical Society held on April 21, 1902.

respectively. Or, since q is an arbitrary distance, let it be replaced by nx_0, where n is zero or any integer and x_0 is an arbitrary distance. Write also for brevity

$$m_0 = \frac{x_0}{2a\sqrt{t}}, \quad m = \frac{x}{2a\sqrt{t}}. \tag{4}$$

Then (3) may be written

$$V = C\int_{nm_0-m}^{nm_0+m} e^{-z^2}\,dz. \tag{5}$$

Now since this satisfies (1) for every value of n, (1) will be satisfied also by the sum of any number of such expressions. Hence a general solution of (1) is

$$V = \sum_{n=0}^{n=\infty} C_n \int_{nm_0-m}^{nm_0+m} e^{-z^2}\,dz. \tag{6}$$

To illustrate the application of this, consider the case of a homogeneous sphere cooling from an initial uniform temperature u_0 above an assumed constant temperature of the surface of the sphere. Then if u be the temperature at a distance r from the center of the sphere at any time t after the epoch, v in (1) may be replaced by $r(u_0 - u)$, x by r, and x_0 by the radius of the surface of the sphere, r_0, say. Thus

$$\frac{\partial r(u_0-u)}{\partial t} = a^2\frac{\partial^2 r(u_0-u)}{\partial r^2}$$

must be integrated subject to the following conditions:

$$ru = ru_0 \quad \text{for} \quad t = 0 \quad \text{and} \quad r > r_0, \tag{a}$$

$$ru = 0 \quad \text{for} \quad r = r_0, \tag{b}$$

$$ru = 0 \quad \text{for} \quad r = 0, \tag{c}$$

$$ru = 0 \quad \text{for} \quad t = \infty. \tag{d}$$

Condition (a) shows that $C_0 = 0$, and condition (b) shows that

$$C_2 = C_4 = C_6 \cdots = 0,$$

$$C_1 = C_3 = C_5 \cdots = \frac{2r_0u_0}{\sqrt{\pi}}.$$

With these values of the constants C_n, equation (6) becomes

$$r(u_0-u) = \frac{2r_0u_0}{\sqrt{\pi}}\left\{\int_{m_0-m}^{m_0+m} e^{-z^2}dz + \int_{3m_0-m}^{3m_0+m} e^{-z^2}dz + \cdots\right\},$$

and this will be found to satisfy the conditions (c) and (d).

ROTARY POLARIZATION MECHANICALLY PRODUCED.[1]

BY ARTHUR W. EWELL.

IN previous papers[2] the author has shown that when plane-polarized light traverses jelly, subjected to twist in a rubber tube, the plane of polarization is rotated in the opposite direction to the twist. Some such result had been looked for in vain by F. E. Neuman and Drion.[3]

The work to be very briefly reviewed in this abstract was performed during the two past years and had as its object a search for definite quantitative relations between the rotary polarization and the twist and twisting moment, and to determine what other conditions besides the twist influence the rotary polarization. Most of the work has been performed with jelly composed of the choicest calves' foot gelatine, water and glycerine; the proportion best adapted to most of this work was 1 g. of gelatine to 5 c.c. of water to 5 c.c. of glycerine. A simple form of biquartz polarimeter was used with mechanical attachments for applying and measuring twists and longitudinal compression and elongation. The polarizer and graduated circle of the analyzer could be rotated together by means of gearing, thus facilitating reading the rotation in different azimuths to eliminate double refraction.

A lateral envelope is not essential, but the rotatory polarization is greatly increased by an elastic envelope. For example:

Jelly cylinder 8 cm. long, 2 cm. diameter—

	No Envelope.		Rubber Tube Slipped Over.	
Twist,	−170°	190	90	−90
Rotation,[4]	1°	−1	− 6	9

Longitudinal compression increases the rotation while longitudinal elongation decreases it. For example:

3.7 cm of jelly in a soft-rubber tube subjected to — 90° twist, decrease of length is taken as positive—

Change of length (mm.),	0	1.4	2.5	−1.6	−3.5	−5.4
Rotation,	14	35	74	6	5	1

Upon plotting changes of length as abscissæ and rotations as ordinates the observations are found to lie quite closely upon exponential curves.

Hydrostatic pressure was applied to jelly twisted in rubber tubes up to 160 lbs. per sq. in. with no observable change in the rotation.

[1] Abstract of a paper presented at the meeting of the Physical Society held on October 25, 1902.

[2] American Journal of Science, August, 1899; Physikalische Zeitschrift, I., No. 18; Johns Hopkins University Circulars, June, 1900.

[3] Verdet, Optique, II., p. 389.

[4] The natural rotation of the jelly has always been subtracted from the observed rotation and the rotations given are the differences due to the twist.

Relaxation effects are very conspicuous but may be eliminated by reading the actual zero of twist before and after each twist. There is a curious influence of the previous history of the jelly; a prolonged preceding twist increases the rotation for an immediately following less twist in either direction but has little influence upon a greater twist. A rapid succession of twists in opposite directions of short duration has no effect. The rotation for a given twist falls off with excessive rise of temperature.

If then, the longitudinal strain is kept constant, if relaxation effects are eliminated, if small twists precede greater, and if the temperature remains sensibly constant, under these conditions we should expect that the rotatory polarization of a given jelly cylinder inside a permanent rubber lateral envelope, for a given twist, would be constant and definite, and such is approximately the case. For example:

3 cm. Length of jelly in rubber tube—

July 21. A. M.

Twist,	−90	90	−110	110	−120	120	−130	130
Rotation,	1	−4	2	−13	2	−18	2	−26
Sum,	5		15		20		28	

July 21. P. M. After Tube has been Removed from Instrument, Turned End for End, and Replaced.

Twist,	60	−60	90	−90	110	−110	120	−120	130	−130
Rotation,	−1	1	−6	0	−13	0	−21	1	−26	1
Sum,	2		6		13		22		27	

July 22. A. M. After Tube has been Again Removed, Turned End for End, and Replaced.

Twist,	−60	60	−90	90	−110	110	−120	120	−130	130
Rotation,	0	0	2	−4	2	−11	3	−17	4	−22
Sum,	0		6		13		20		26	

The dissymmetry in the two directions is largely due to dissymmetry in the rubber tube. The sum or average of the rotations is found to be quite closely proportional to the fourth power of the twist.

The mechanical properties of the jelly were carefully studied. The most interesting discovery was that the rigidity is greatly increased by longitudinal elongation. The method of vibration was used, timing the period of vibration of a clamp attached to the lower end of a jelly cylinder, loaded with different weights. The cross-section was determined by the displacement of a known length of jelly in benzine, correction being made for the change in cross-section when the jelly was stretched. The rigidity of a certain jelly cylinder was found to be 1.8×10^5 when the length was 15.7, 2.5×10^5 when the length was 16.2 and 7.6×10^5 when the length was 16.9.

The author has also formulated a theory to explain the general phenomena, a full statement of which would be too long for this abstract.

The points successively disturbed by two opposite circularly polarized rays, parallel to the axis of a twisted cylinder, lie along two opposite spirals. Since the stresses and strains increase as one leaves the axis of a twisted cylinder, conditions on the side of these spirals farther from the axis will have a determining influence. The twist will produce elongation along the outside of the spiral of the same direction of twist as the cylinder, which will belong to the circularly polarized ray of opposite sign, and compression along the outside of the other spiral. Since in general decrease in the velocity of light accompanies increase in density or longitudinal compression, we should expect that the ray circularly polarized in the opposite direction to the twist of the cylinder would be propagated faster or the plane of polarization would be rotated in the opposite direction to the twist. The positions of the axes of strain are such that longitudinal compression increases the compression along one spiral more than it decreases the elongation along the other and hence should increase the rotation.

WORCESTER POLYTECHNIC INSTITUTE,
November 10, 1902.

THE PRESSURE DUE TO RADIATION.[1]

BY E. F. NICHOLS AND G. F. HULL.

AN account of the results of later measurements of radiation pressure in comparison with measurements of radiant energy was communicated. The principal change in the method of measurements was the substitution of a silver disc for the bolometer earlier used to measure the energy of the beam. Pressure measurements were carried through for a beam which had traversed a plate of red glass and one which had passed through a water cell in addition to a beam as employed in the preliminary work.[2] No difference in the relation of pressure to energy depending on the wave-length of the incident beam was discovered.

The later measurements were far more comprehensive and accurate than the earlier ones had been. Many corrections to the final results were made after a careful and exhaustive study of all the conditions affecting the experiments. The results are in agreement with the Maxwell-Bartoli theory to within about one per cent. A study of the degree of accuracy attained in the various kinds of measurements entering into the results shows that the probable error of the whole is not greater than one part in one hundred. A detailed account of the experiments will be found

[1] Abstract of a paper presented at the meeting of the Physical Society held on December 31, 1902.

[2] E. F. Nichols and G. F. Hull, PHYS. REV., 13, p. 307, 1901.

in the forthcoming Proceedings of the American Academy of Arts and Sciences for 1902–3. More condensed accounts will appear elsewhere. A demonstration of the effect of radiation pressure upon the torsion balance used in the experiments was made before the society.

A vacuum tube in the form of an hour-glass containing calcined puff-ball spores mixed with emery sand, was exhibited to illustrate the application of light pressure to the repulsion of comets' tails by the sun. In the preparation of the tube the exhaustion by a good Geissler mercury pump was continued for two days and after sealing off the tube from the pump, mercury in a connected flask was frozen for an hour in solid carbon dioxide and ether. Finally the mercury flask was sealed off from the hour-glass. It was thought that the traces of mercury vapor in the system might thus be frozen out and a very high vacuum obtained. The hour-glass was heated to the point of softening during the entire pumping and freezing process. When the glass was placed in an upright position, and the concentrated beam from an arc light directed horizontally on the falling stream of particles just below the neck of the glass, the small spores were driven off to one side in the direction of the beam while the sand was not deflected from its vertical path.

A hasty computation of the magnitude of the deflecting angle of the spores, due to radiation pressure, was made from the energy of the beam, roughly known, and the size of the spores. The observed deflection was approximately equal to that predicted by computation and the writers believed, for a time, that the forces due to the action of the residual gas constituted only a fraction of the total deflecting force. A later review of the computation disclosed an error in the numerical work which had made the predicted deflection due to radiation pressure much too large. The conclusion, therefore, was that the action of the residual gas in the tube considerably outweighed the pressure due to radiation. The writers purpose to make further experiments in which it is hoped the gas effects may be greatly reduced and the radiation pressure remain alone evident.

The Magnetic and Electric Deviation of the Easily Absorbed Rays from Radium.[1]

By E. Rutherford.

THE greater portion of the energy radiated by the natural radioactive substance is in the form of α or easily absorbed rays. Two other types of rays are also emitted, viz.: the β rays, readily deviable by a magnetic field and similar to high velocity cathode rays, and the γ rays non-deviable by a magnetic field, and very penetrating in character.

[1] Abstract of a paper presented at the meeting of the Physical Society held on December 31, 1902.

By employing strong magnetic fields and arrangements to detect a minute curvature of the path of the rays, it was found possible to show that the α rays are, like the β rays, completely deviable by a magnetic field. The deviation is in the opposite sense to the cathode rays so that the rays consist of positively charged particles projected with great velocity.

The general method employed was to pass the rays from the active substance (radium chloride activity 19,000) through a number of narrow slits placed in parallel and to observe the effect of a magnetic or electric field on the ionization produced by the issuing rays by means of a special gold leaf electroscope.

A current of hydrogen gas passed through the electrometer and through the slits in order to prevent the diffusion of any emanation into the testing vessel.

The rays were also found to be partly deviated by passing through a strong electric field.

By combining the results of the magnetic and electric deviation in the usual way it was found that

$$V = 2.5\ 10^9 \text{ cm. per sec.}$$

$$\frac{e}{m} = 6.10^3$$

where V is the velocity of projected particles, e the charge on the particle and m its mass. These results are only approximate and merely indicate the order of the results obtained.

The rays thus consist of positively charged bodies projected with a velocity about one tenth that of light and of mass of the same order as the hydrogen atom and large compared with the electron. The α rays are thus very analogous to the "Canal Strahlen," which Wien has shown to consist of projected particles, atomic in size, carrying with them a positive charge.

The projective nature of the α rays offers a satisfactory explanation of the characteristic properties of these rays. Since all the active bodies and also excited bodies give rise to α rays, it seems probable that they consist in all cases of heavy positive particles projected with great velocity.

The projection of α rays is probably the underlying cause of the series of chemical changes occurring in radioactive bodies, for each change is accompanied by the projection of positively charged bodies.

This continuous and spontaneous emission of heavy bodies from the natural radioactive substances must cause a gradual diminution in their weight.

A Penetrating Radiation from the Earth's Surface.[1]

By E. Rutherford and H. L. Cooke.

IT has been shown by Rutherford (Phys. Zeit., 1902) that the radiations from the naturally radioactive bodies and also the excited radiations include some rays of an extremely penetrating character, which are able to pass through great thicknesses of matter. Since the excited activity obtained from the atmosphere is very similar in character to the excited radiations from thorium and radium, it was thought possible that some penetrating rays might be given off from the surface of the earth and walls and rooms on which excited activity from the air is distributed.

In order to test this point, the amount of ionization was observed in testing vessels of about 1 liter capacity. The method used by C. T. R. Wilson in his experiments on the "spontaneous" ionization of air was employed. The rate of discharge of a well insulated gold-leaf system served as a measure of the ionization. The effect of placing metal screens outside the testing vessel was observed. Screens of thickness of 2 mm. of lead had very little effect on the rate of discharge. A thickness of 5 cm. of lead, however, was found to cut down the rate of discharge by 30 %. A greater increase of thickness had no effect, although 5 tons of pig lead was placed around the apparatus. A thickness of 5 cm. of iron and 70 cm. of water also cut down the rate of discharge about 30 %.

On removing the screens the discharge returned to the original value. These results show that about 30 % of the ionization inside a closed vessel is due to an external radiation of great penetrating power. This radiation appears to come equally from all directions and is probably due to excited activity on the surface of the room in which the observations were made.

These effects could not be due to the presence of thorium or radium in the laboratory, for similar results were observed in the library which was free from all possible contamination by radioactive substances.

A decrease was also observed with metal screens when the testing vessel was placed down on the frozen earth at some distance from the laboratory. Under the conditions of the experiment, the effect was not quite so marked as inside the laboratory.

In brass vessels which had been thoroughly cleaned the rate of discharge corresponded to a rate of production of 10 ions per c.c. per second. This number was reduced to about 6 by the addition of a thick metal screen. This value is much lower than the value 19 found by C. T. R. Wilson in a glass vessel silvered on the inside.

[1] Abstract of a paper presented at the meeting of the Physical Society held on December 31, 1902.

SOME EXPERIMENTS ON THE ELECTRICAL CONDUCTIVITY OF ATMOSPHERIC AIR.[1]

BY J. C. MCLENNAN AND E. F. BURTON.

I. INTRODUCTION.

IN a paper by H. Geitel[2] reference is made to a gradual increase observed in the conductivity of a mass of atmospheric air after being confined in an air-tight chamber. This effect was found to require from four to five days to reach its maximum value and was observed in localities where no thorium compounds or other known radioactive substances existed.

In a subsequent investigation Elster and Geitel[3] found that the air which had been confined for some time in closed caves or house cellars possessed an abnormally high conductivity. This phenomenon, together with the observed increase in conductivity mentioned above, they concluded, could not be due to the presence of dust or water vapor. They traced it rather, in both cases, to the existence of some undetermined radioactivity in the confining walls.

More recently these physicists discovered that atmospheric air possessed the property of exciting induced radioactivity in bodies exposed under negative electrification. This phenomenon of induced or excited radioactivity had been previously observed by Rutherford in bodies exposed to air drawn from the neighborhood of thorium compounds and had been connected by him very directly with an emanation which these salts emit. This emanation he found possessed the property not only of exciting radioactivity in all solid substances in its neighborhood, but also of ionizing any gas with which it was in contact.

Since atmospheric air has been shown by Elster and Ge tel,[4] C. T. R. Wilson[5] and others to be continually ionized by some agent and since it has also been shown to possess the property of exciting radioactivity one is forced to conclude there is present in the air an emanation possessing properties similar to that emitted by thorium compounds.

Hitherto the source of such an emanation has not been determined, but, as the phenomena of induced radioactivity and spontaneous ionization universally characterize atmospheric air, it seems evident, since thorium compounds are but sparsely distributed in nature, that sources other than these must exist.

[1] Abstract of a paper presented at the meeting of the Physical Society, held on December 31, 1902.

[2] Phys. Zeit., 2, pp. 116–119, 1900.

[3] Phys. Zeit., 2, pp. 560–563, 1901.

[4] Loc. cit.

[5] Proc. Roy. Soc., March, 1901.

Recalling the experiments of Elster and Geitel, it seems probable that the earth's surface, and possibly too the materials used in the construction of their apparatus, are sources of this emanation. As but little evidence existed in favor of this conclusion, the writers recently made a series of observations upon atmospheric air confined in air-tight vessels of different metals. The result of the investigation showed that the conductivity of the enclosed air depended very largely upon the material of which the receiver was made, and the effects observed would seem to indicate that all metals in varying degree are the sources of a marked though feeble radioactive emanation.

II. Apparatus.

In these observations the air whose conductivity was to be measured was confined in a cylinder 125 cm. in length and 25 cm. in diameter, similar to that shown in Fig. 1. In the first experiments it was made of sheet iron coated with zinc, and in the later experiments linings of various metals were inserted in order to examine their effect upon the conductivity of the enclosed air. The bottom and cover were removable, and, when in position, were made air-tight by means of cement. Into a flanged opening in the cover was fitted an ebonite plug about 5 cm. in diameter. A brass tube B was passed through this and into it a second ebonite plug was tightly fitted. This second plug carried a brass rod C which extended almost to the bottom of the cylinder. The brass tube B, which was earthed throughout the measurements, served as a guard ring and prevented any leak from the vessel to the rod C across the ebonite plugs.

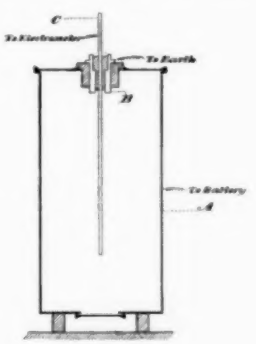


Fig. 1

The conductivity was measured by placing the cylinder upon an insulated platform, charging it by means of a set of small storage cells to a potential of 165 volts which sufficed for the saturation current, and observing the rise in potential of the electrode D which was joined to a quadrant electrometer in the usual manner. The sensibility of the electrometer was such as to produce a deflection of 1,000 mm. on a scale above meter distance for a potential difference of one volt between the quadrants.

III. Conductivity Measurements — Time Effects.

Before enclosing air for examination the cylinder was placed in an open window in the laboratory, with the ends removed and the air allowed to blow through it for some time. The top and bottom were then replaced,

cemented in position and the cylinder connected with the electrometer as quickly as possible.

Measurements on the conductivity were made at intervals of a few minutes at first, and it was invariably found that a rapid decrease in the ionization took place until a minimum value was reached. The conductivity then slowly increased and approached a limiting value in the course of two or three days.

TABLE I.

Current: Arbitrary Scale.

Time.	Current: Arbitrary Scale.	
	Pressure — 501 cm.	Pressure — 74.2 cm.
Column I.	Column II.	Column III.
10 minutes.	30.0	17.1
15 "		9.3
30 "	17.5	6.2
50 "	13.2	5.5
1 hour.	12.0	
1.25 "		5.13
1.5 "	9.9	
1.75 "	8.9	5.1
2.00 "	8.0	4.9
4.00 "	6.6	4.8
5.00 "	7.6	
6.00 "		5.0
10.00 "		5.8
22.00 "	19.2	
25.00 "		6.0
29.00 "		6.0
32.00 "	22.0	
34.00 "		6.3
44.00 "	24.0	
45.00 "		6.5

In repeated tests carried out in this manner with the zinc cylinder it was found that, while the initial conductivity varied from day to day, there was always observed a rapid decrease to a constant minimum followed by a gradual rise to a constant limiting value.

A typical set of values for the conductivity of air confined in the zinc cylinder at atmospheric pressure are given in column III. of Table I., the currents being expressed in arbitrary units, and the times being taken from the closing of the cylinder.

The ionization curve for these values, Fig. 2, shows that the minimum current, 4.8, was reached in about four hours after the air was enclosed.

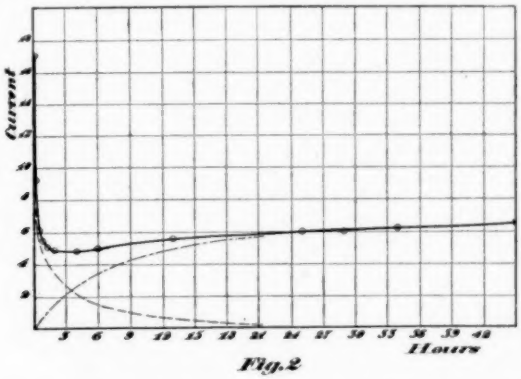


Fig. 2

After about eighteen hours the curve indicates that the conductivity was tending towards a limiting value, which the reading taken after forty-four hours showed to be about 6.5.

As a variation in the experiments a series of tests were made in air confined in a receiver at high pressures. The cylinder in this case was of heavy rolled iron coated with zinc and was of the same dimensions as that used in the first measurements.

The results of observations on the conductivity of air confined at a pressure of about seven atmospheres are given in column II., Table I., the scale used being the same as before. The curve representing these values is shown in Fig. 3, and exhibits the same characteristics as that for the lower pressure.

We have again the rapid decrease to a minimum followed by a gradual rise tending towards a limiting value. The minimum conductivity in this case was about 6.6, and was reached in about four hours after the required pressure in the cylinder had been obtained. The time occupied in pumping the air was about one hour.

In seeking for an explanation of the curves shown in Figs. 2 and 3, their twofold origin, as indicated by the dotted lines, is at once suggested, the conductivity in the initial stage being due to an agent subject to rapid decay, and that in the second to one whose power shows a gradual increase.

The first of these dotted curves is similar to that given by Rutherford for the conductivity of the air in a chamber which had been cut off from a second containing thorium oxide after the two had been in communication for some time, while the second is similar to that given by him for the increase in the conductivity of the air in one chamber when placed in communication with another containing thorium oxide.

It will thus be seen that the first portion of the curves in Figs. 2 and 3 can be explained upon the supposition that a radioactive emanation, probably having its origin in the earth's surface, was introduced into the cylinder with the air, the decay of this emanation being the cause of the decrease in the conductivity, and the second portion upon the supposition that a radioactive emanation is given off by the walls of the containing vessel. On this view the limiting value to which the conductivity curves tend would represent a condition of equilibrium, where the rate of decay of the emanation was equal to the rate at which it was produced.

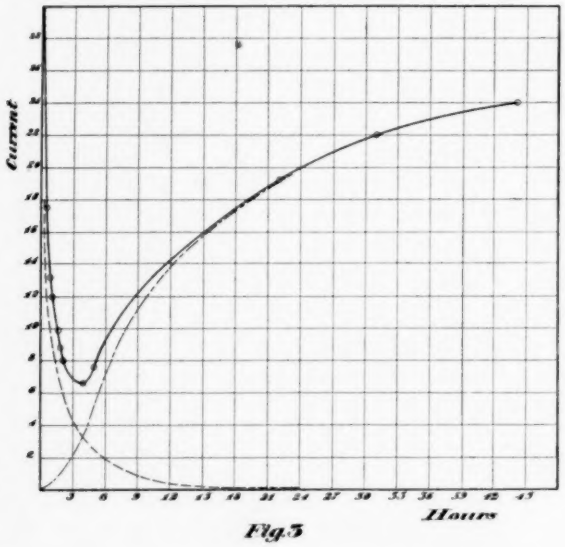


Fig. 3

As both the low and the high pressure cylinders were made of the same material and were of the same size, one would expect the same amount of the emanation to be present in both when the steady state was reached. With an easily absorbed radiation from this emanation, we should obtain a limiting conductivity independent of the pressure. But, since a very great difference was found in the limiting conductivities at the two pressures, it would appear that the radiation possesses considerable power of penetration and is not easily absorbed.

The difference in the initial conductivities given in columns II. and III., Table I. may also be readily explained by the difference in the air pressures. The time required to fill the high pressure cylinder and the decay taking place during that time in the emanation introduced with the air, preclude a comparison of the amounts of active emanation present in each cylinder when the first observation upon their conductivities was made; but, if the amount in the high-pressure cylinder were equal to or

greater than that in the low-pressure cylinder, the difference in the initial conductivities is explained, while, if it were less, the greater density of the air in the high-pressure cylinder and the consequent greater absorption would still account for the higher conductivity.

IV. Effect of Different Metals.

To ascertain whether the conductivity of the air enclosed was affected by a change in the metal composing the walls of the receiver, linings of tin and lead were in turn fitted into the zinc cylinder used in the first experiment.

Tests of the conductivity were made both with and without the tin and lead linings. Before each test the cylinder was well aired and sealed in the manner already described. As soon as the air was enclosed measurements on the conductivity were begun and continued at stated intervals as before.

Table II.

Time.	Current; Arbitrary Scale.		
	Zinc.	Tin.	Lead.
10 minutes.	13.0	13.0	15.4
15 "	8.3	11.0	
30 "	5.4	7.6	13.4
45 "	5.3	7.3	12.1
1.0 hours.	5.0		11.55
1.5 "	4.85	7.2	10.8
2.0 "		7.3	11.5
2.5 "		7.3	
5.0 "	5.0		11.8
7.0 "	5.3		11.9
9.0 "			12.0
12.0 "		7.9	
18.0 "	6.0	8.0	

The values obtained for the conductivity with each of the metals are given in Table II. and curves representing these values are shown in Fig. 4.

The curves for the different metals, it will be seen, have the same characteristics. In each there is a rapid drop to a minimum and a gradual rise towards an ultimate limiting value. It is interesting to note that a considerable difference was found in the minimum conductivities for the three metals and that the final limiting values also varied.

The decay of an emanation introduced into the cylinders with the air would again account for the first portion of the curves; a radioactive emanation from the metallic walls would explain the existence of the

second portion, while the differences in the minimum and limiting values may be considered to have their origin in variations in the rate at which the emanation is given off by the different metals.

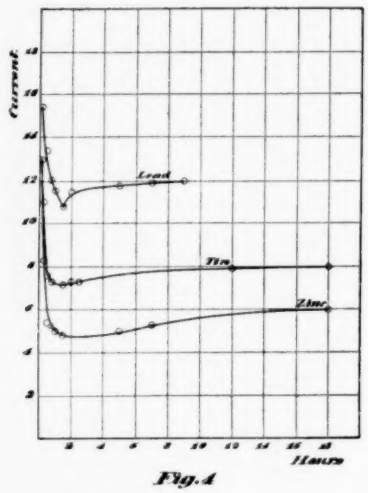


Fig. 4

In this connection it will be noted that the limiting values of the conductivities range according to the atomic weights of the metals, lead having the highest, tin the next and zinc the lowest.

V. Effect of Variations in Pressure.

In order to investigate the relation between the conductivity of air and the pressure at which it was confined, the heavy cylinder was filled to a pressure of about seven atmospheres and allowed to stand for some days until its conductivity assumed a constant value.

Table III.

Pressure in cm.	Current; Arbitrary Scale.	Pressure in cm.	Current; Arbitrary Scale.
501.0	23.5	69.3	5.8
481.0	22.7	62.0	5.4
420.0	21.2	53.0	4.7
384.0	20.3	44.2	4.2
272.0	15.8	35.0	3.5
227.0	14.1	22.4	2.7
176.0	11.7	18.2	2.2
125.0	9.3	14.0	1.8
74.2	6.5	4.4	1.1

The air was then allowed to escape gradually and the pressure reduced from 501.0 cm. to 4.4 cm. of mercury, the conductivity being measured at these and at various intermediate pressures. The results are given in Table III. and the conductivity curve in Fig. 5.

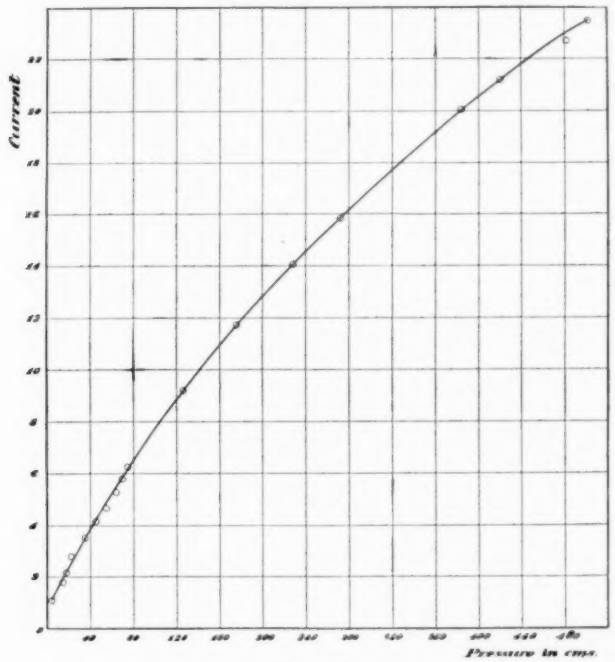


Fig. 5

The ionization curve so nearly approaches a straight line that we may almost conclude, in view of the wide range of the pressures examined, that the conductivity was proportional to the pressure. This result is what we should expect to obtain with an emanation maintained at a constant strength and emitting a radiation of a penetrating nature.

VI. Penetrating Rays from External Sources.

While the effects described up to the present may be explained by supposing the ionization to be caused by a radioactive emanation sent off from the metals, it has been found that part of the conductivity cannot be accounted for in this way, but must be attributed to an active agent external to the receiver.

The heavy cylinder referred to above was filled with air to a pressure of about 400 cm. of mercury and allowed to stand until its conductivity

had become steady. It was then placed in an insulated galvanized iron tank, 150 cm. in height and 75 cm. in diameter, which was gradually filled with water so as to surround the cylinder with a layer 25 cm. in thickness. The initial conductivity before the water was admitted was 21.1. As the water rose the conductivity decreased and fell to 13.3 when the tank had been filled. The values for the conductivity during the experiment are given in Table IV., and they show that the loss was almost directly proportional to the rise of the water. The total fall in conductivity, it will be seen, was about 37 per cent.

TABLE IV.

Depth in cm.	Current; Arbitrary Scale.	Depth in cm.	Current; Arbitrary Scale.
0.0	21.1	97.0	14.3
15.0	20.5	110.0	13.75
60.0	18.0	120.0	13.3
77.0	17.0		

The experiment was repeated with the cylinder placed in a tank 50 cm. in diameter. This tank permitted the cylinder to be surrounded by a layer of water 12.5 cm. in thickness, and it was found when the water was poured in that the conductivity fell off 17.5 per cent.

From these results it is evident that the ordinary air of a room is traversed by an exceedingly penetrating radiation such as that which Rutherford has shown to be emitted by thorium, radium and the excited radioactivity produced by thorium and radium.

In order, therefore, to reach definite conclusions regarding the extent and true character of the effect of various metals upon the conductivity of the air which they enclose, it will be necessary to cut off the enclosing vessel entirely from the action of this external radiation, and the writers have not yet carried their experiments to this point.

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